

Spatial track: range acquisition modeling



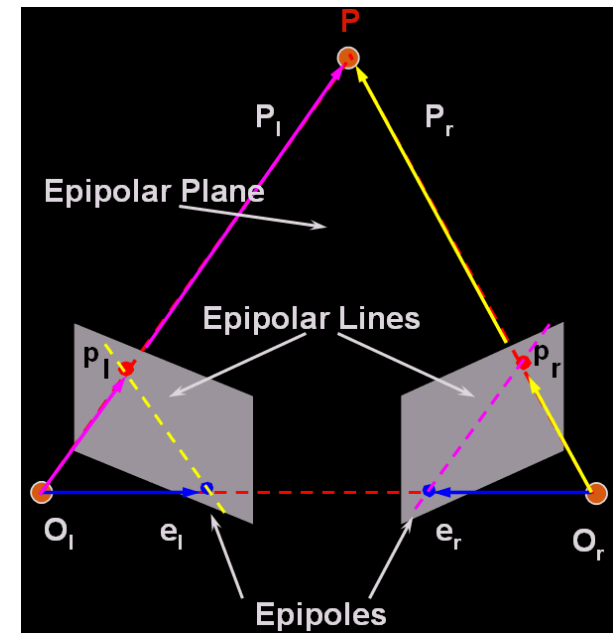
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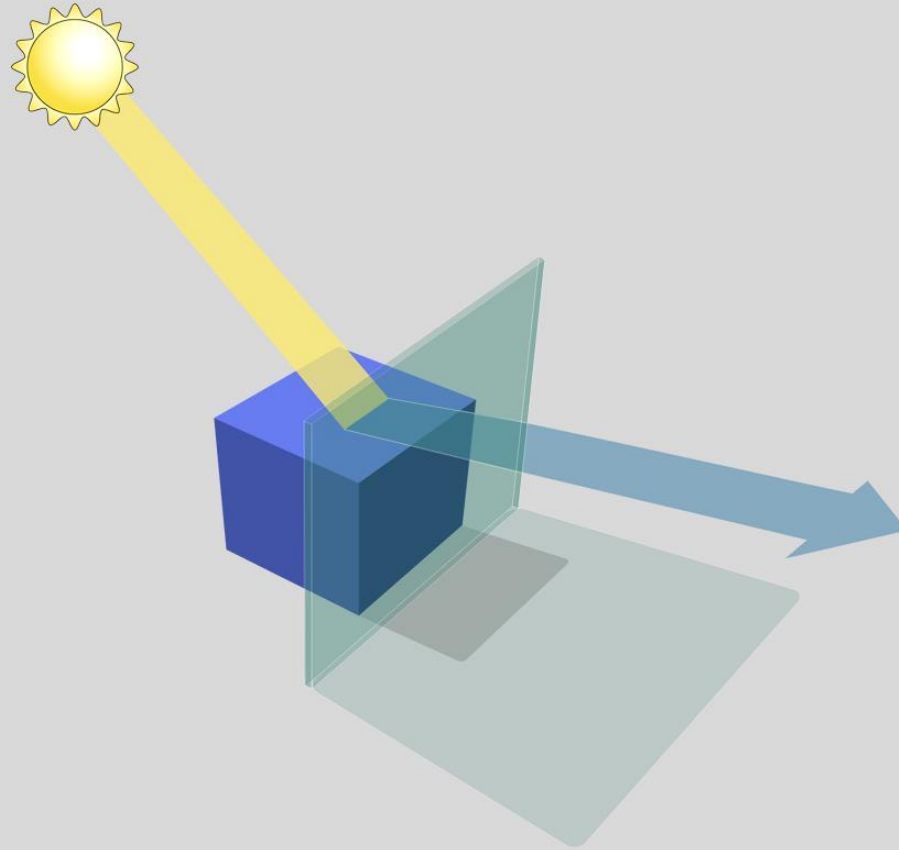
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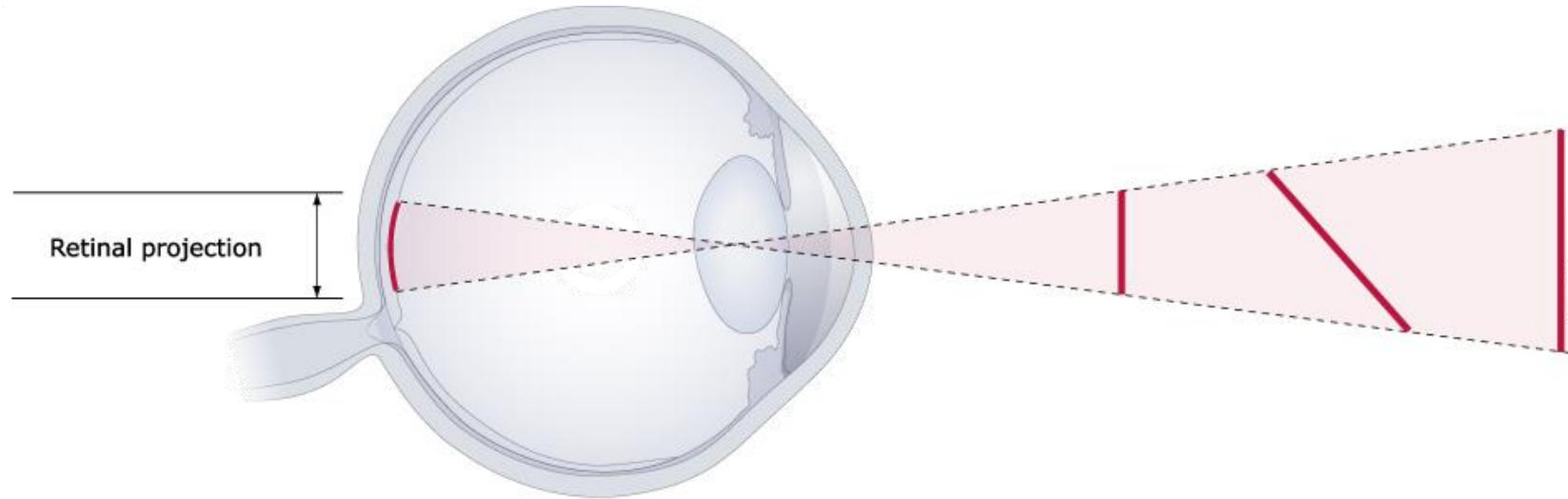
<http://vision.unipv.it/CV>



The inverse problem



Physical space geometrical properties: distances in depth - the inverse problem

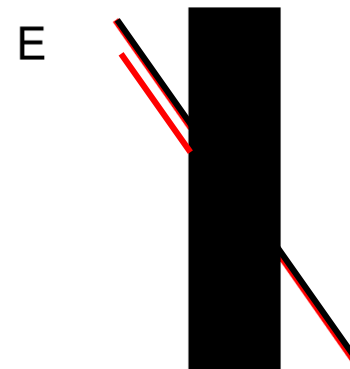
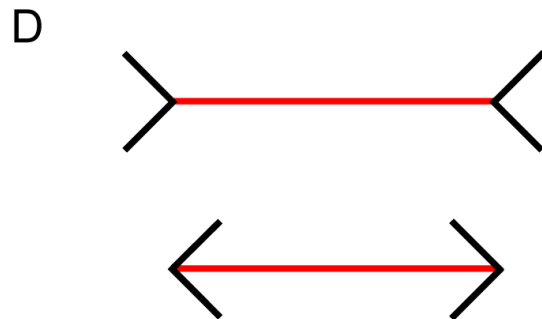
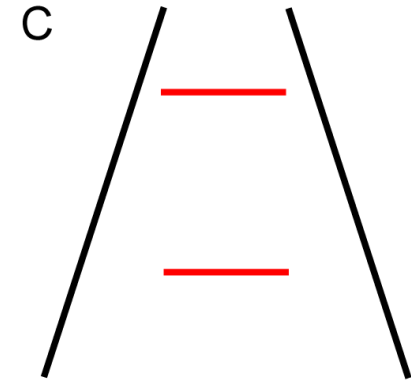
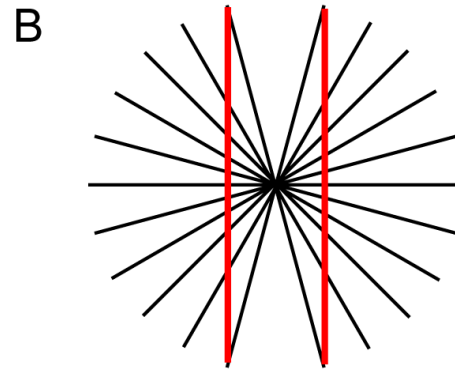
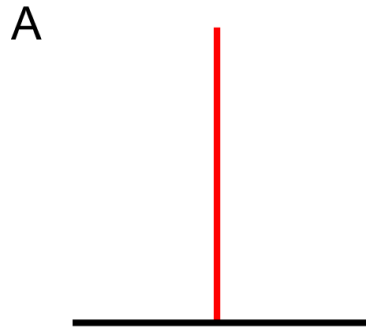


- The stimuli produced when energy interacts with sensory receptors cannot specify the real-world sources of that energy
- To survive, animals need to react successfully to the sources of the stimuli, not to the stimuli as such
- This quandary is called the inverse problem

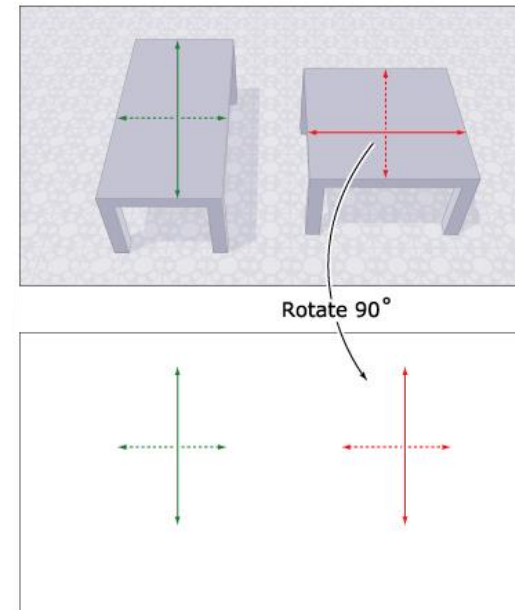
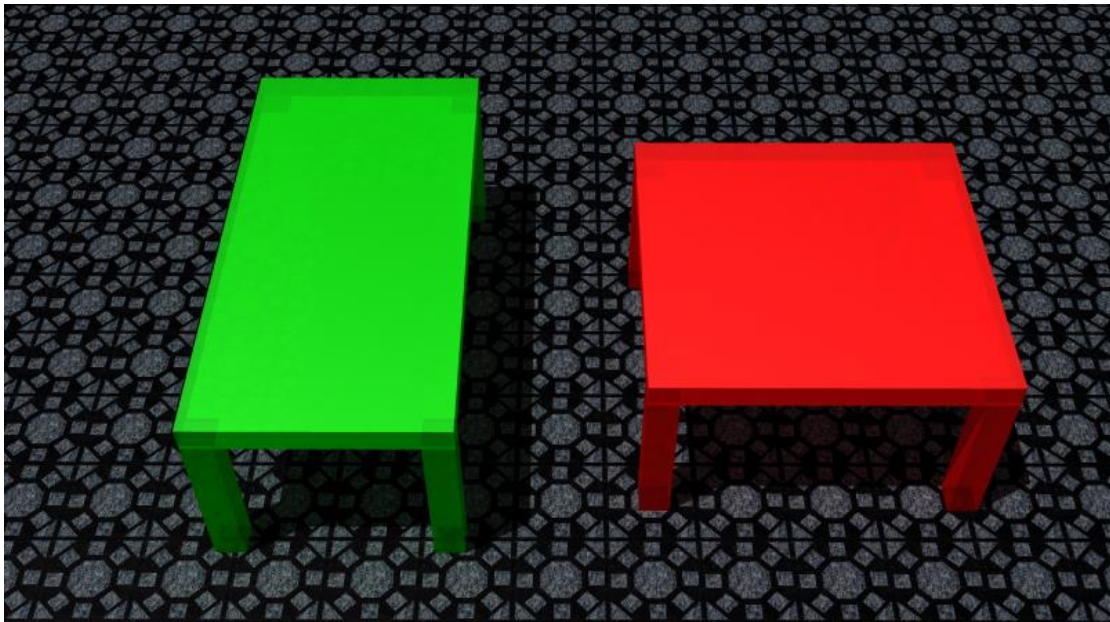
Explanation of Visual Processing and Percepts

- The basic problem understanding vision is that the **real-world sources of light stimuli cannot be known directly**
 - The visual system generates percepts **entirely on the basis of past experience**; stimulus patterns trigger percepts as reflex responses that have been **empirically successful**.
 - Physical **space** is characterized by geometrical properties such as line lengths, angles, orientations and depth distances
 - Our intuition is that the **subjective qualities** arising from these properties should be a more or less direct transformation of physical space
 - As in the domains of brightness and color, however, there are many **discrepancies between measurements of physical space and the geometries people actually see**
-

Physical **space** geometrical properties: line lengths

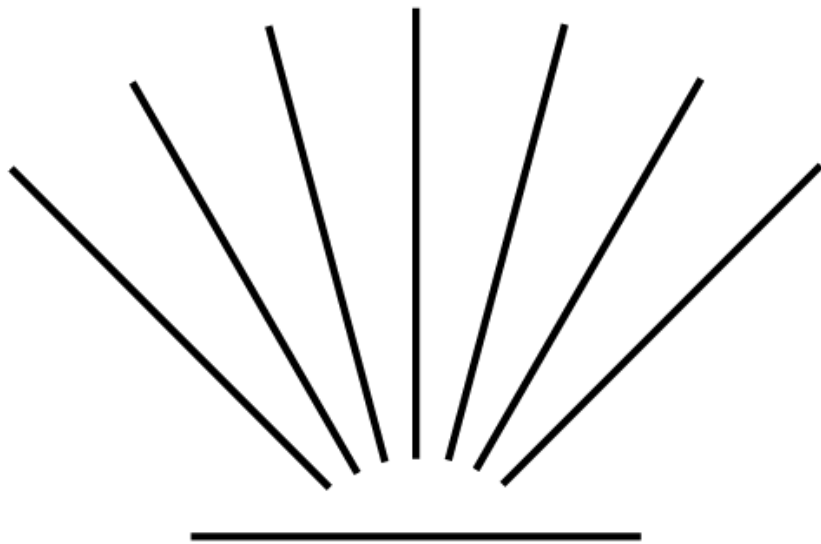


Physical **space** geometrical properties: orientation anisotropy

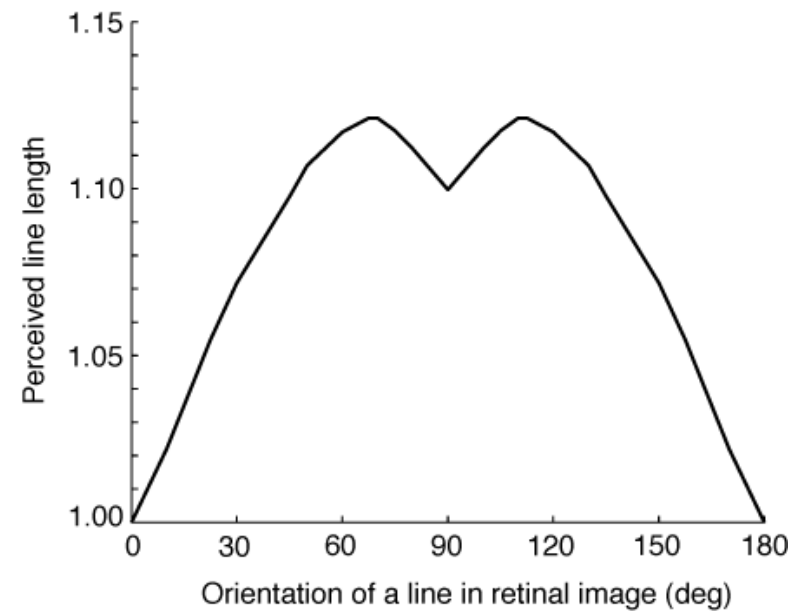


Physical **space** geometrical properties: line lengths

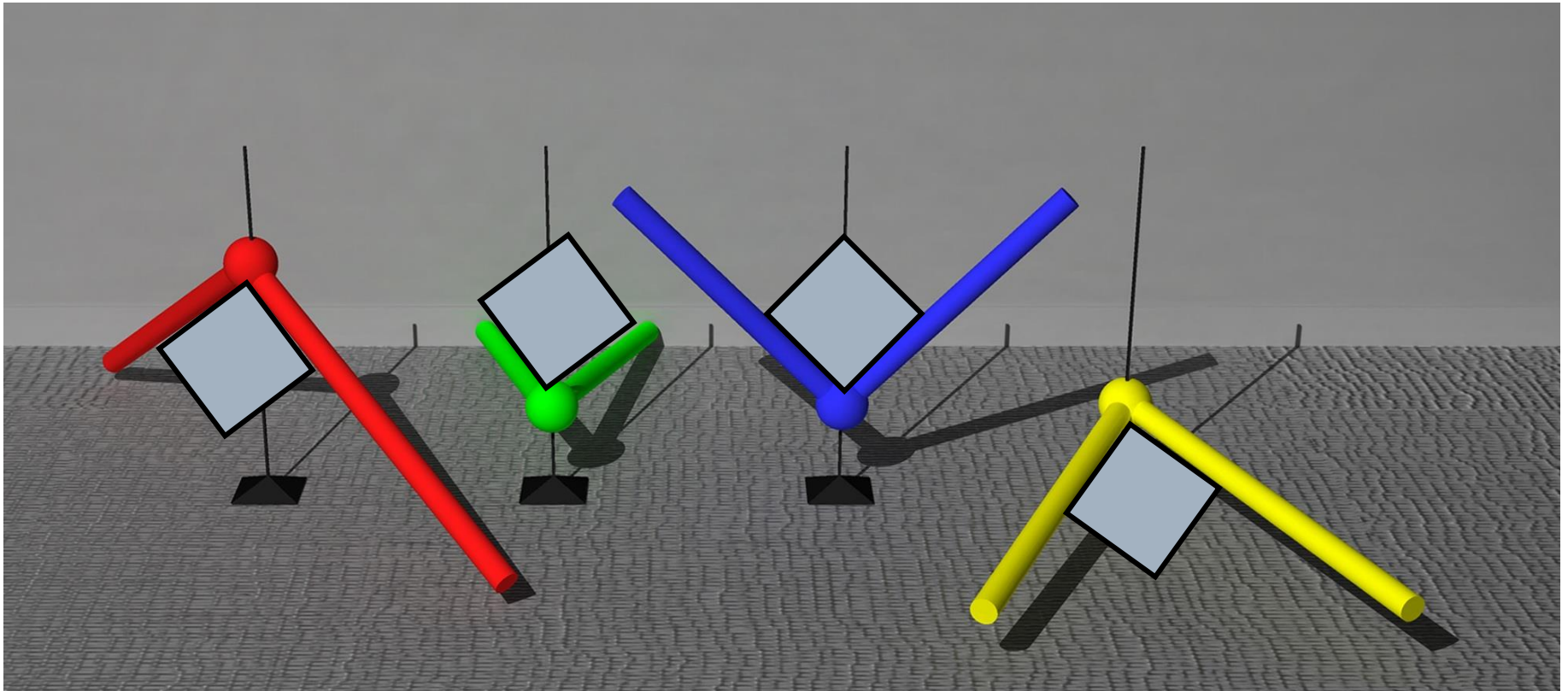
A



B

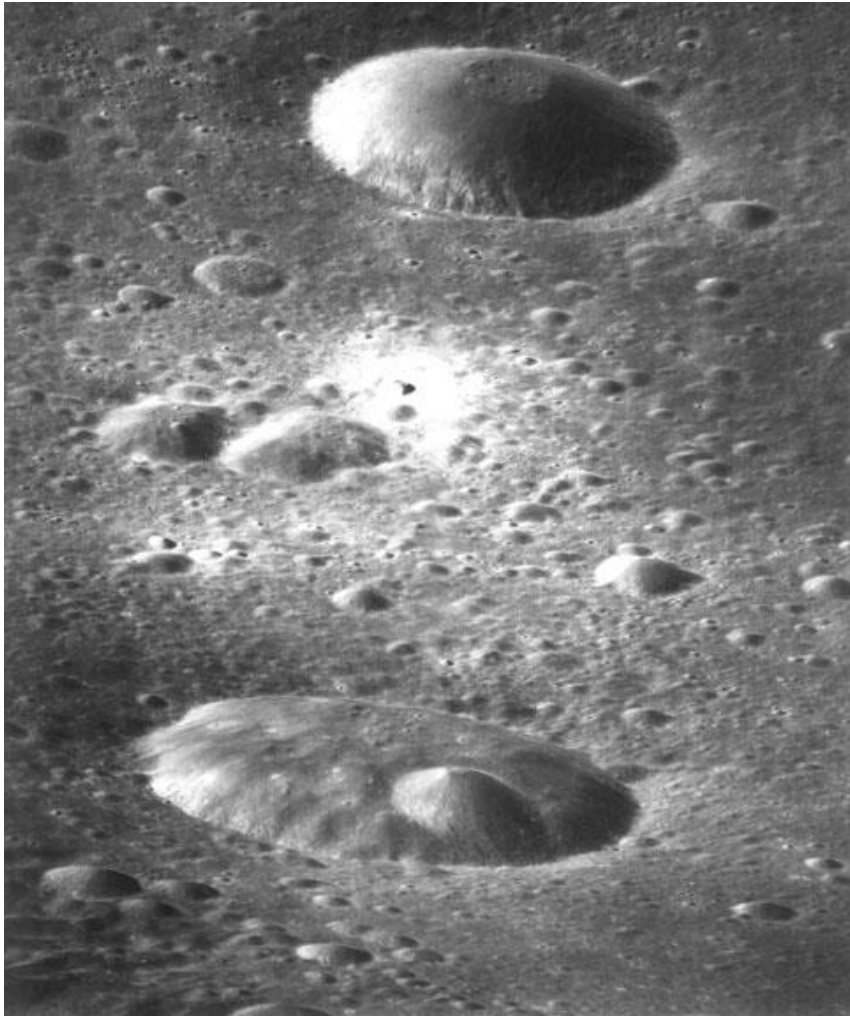


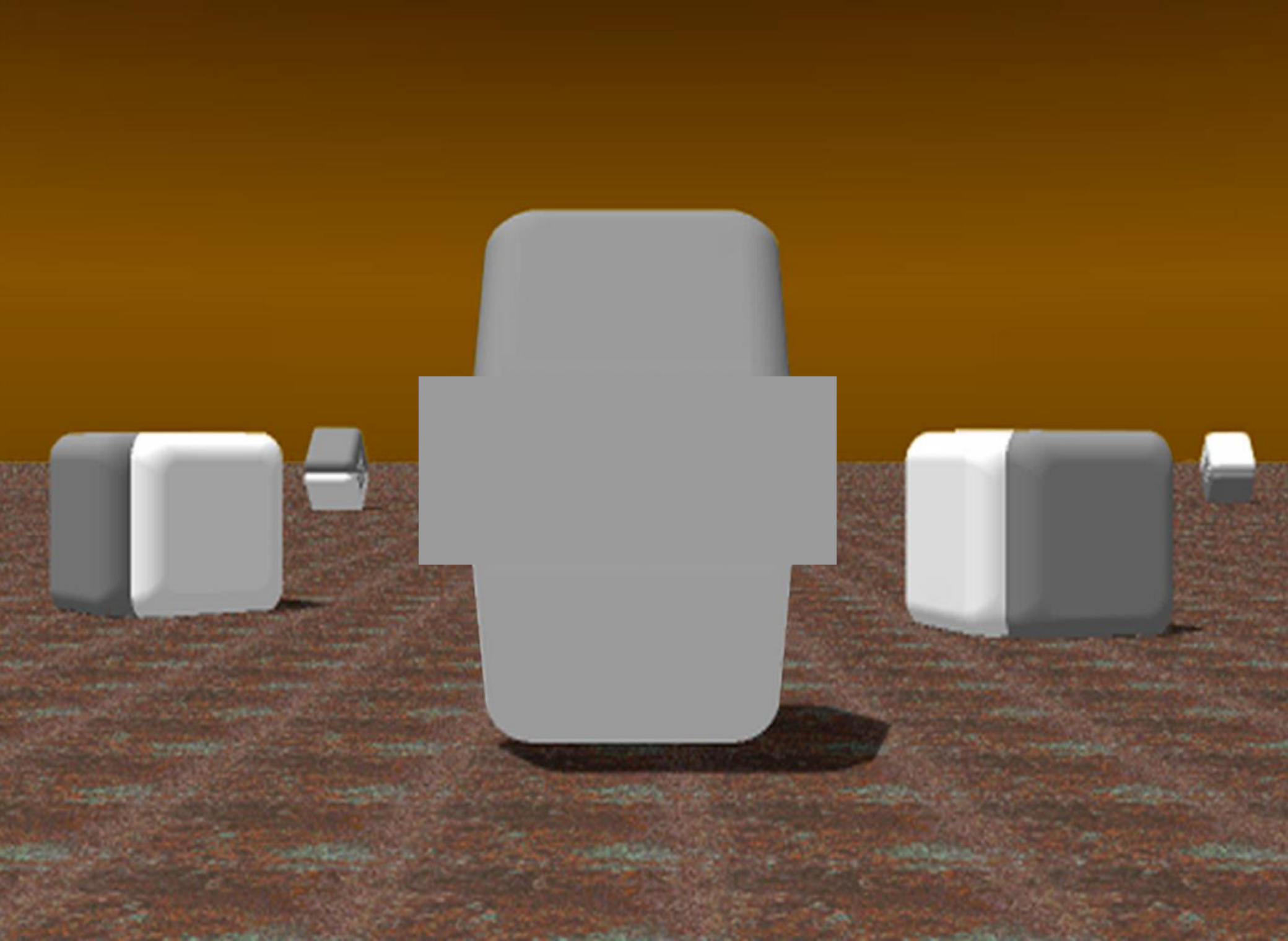
Physical **space** geometrical properties: angles



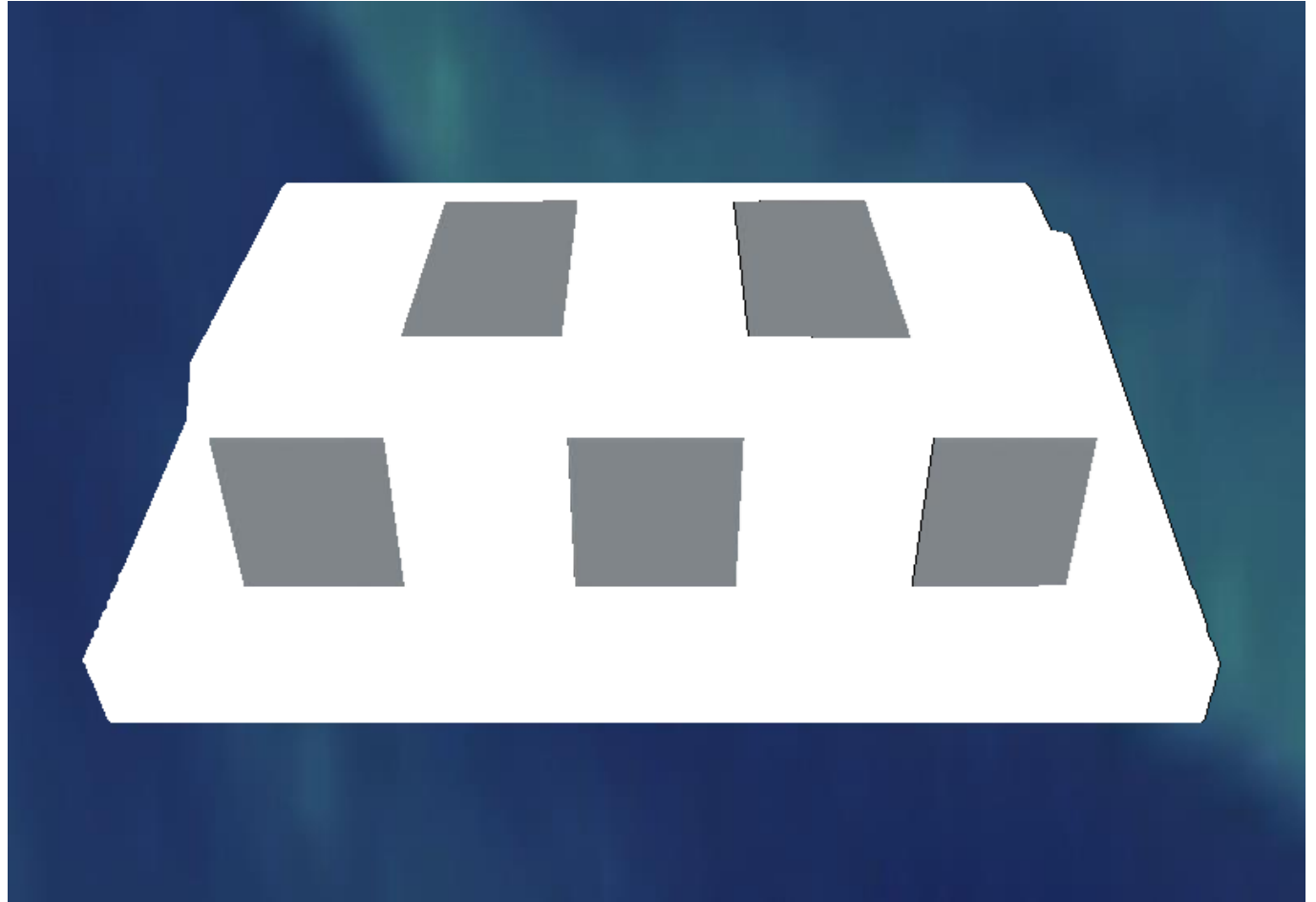
© Dale Purves and R. Beau Lotto 2002

Vision is Inferential: Prior Knowledge



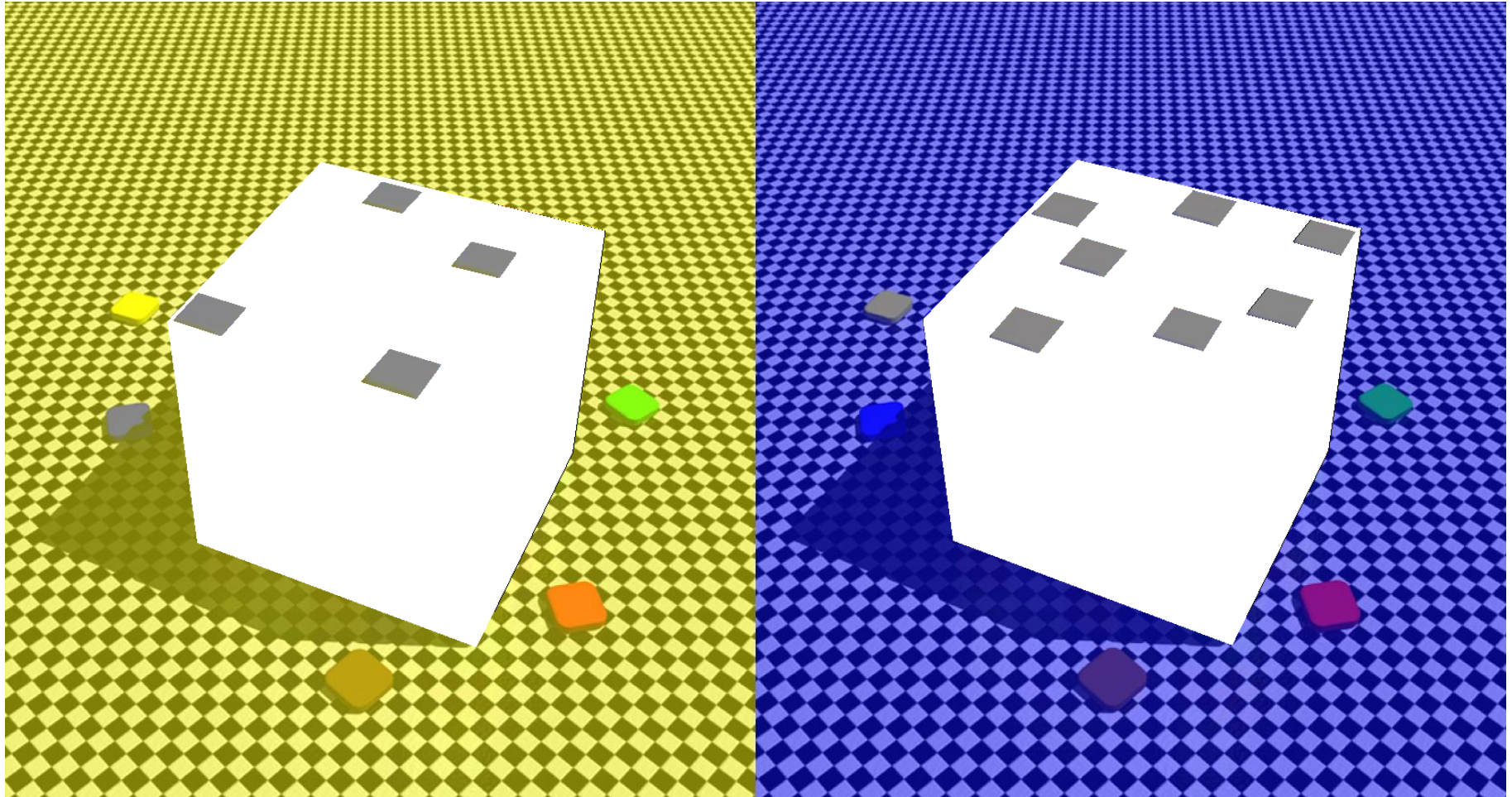


Optic illusions

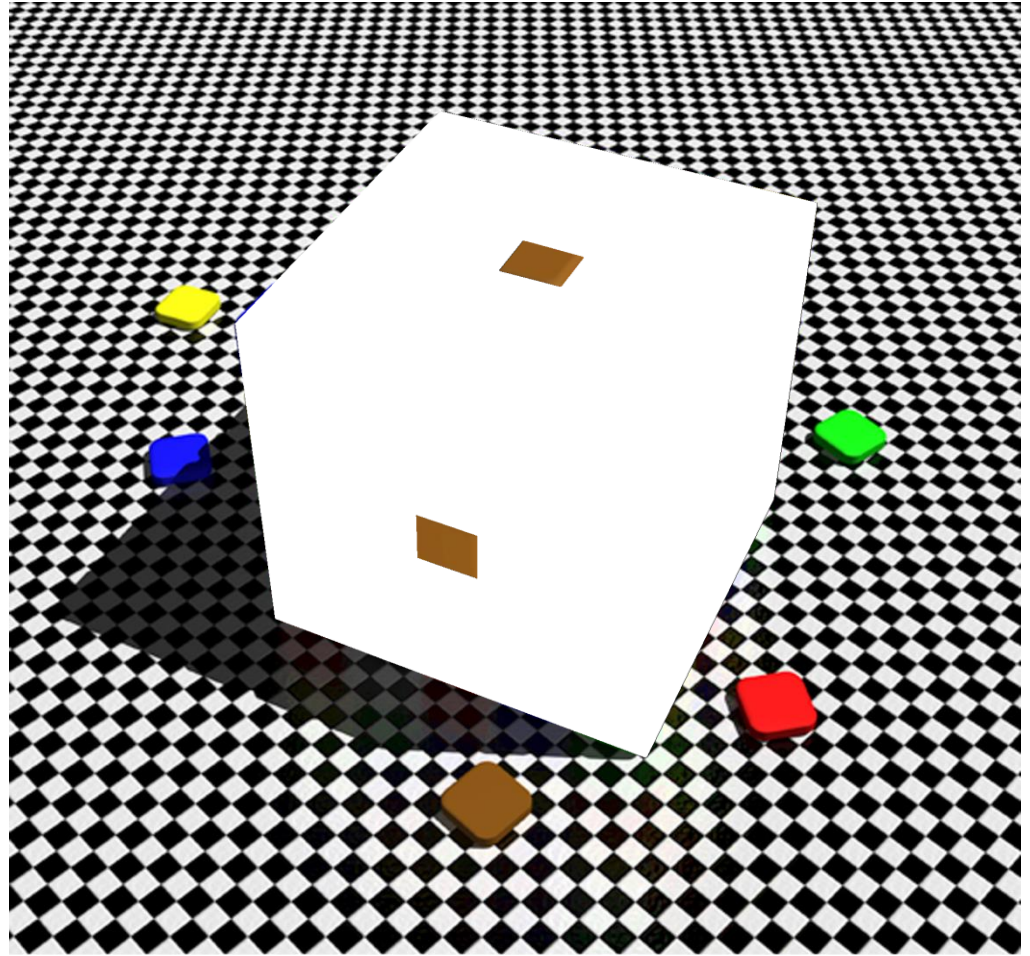


Dale Purves, Cognitive Neuroscience, Duke University

Optic illusions



Optic illusions



Visual cues - The human headway

Overlapping objects

Quantized scenes

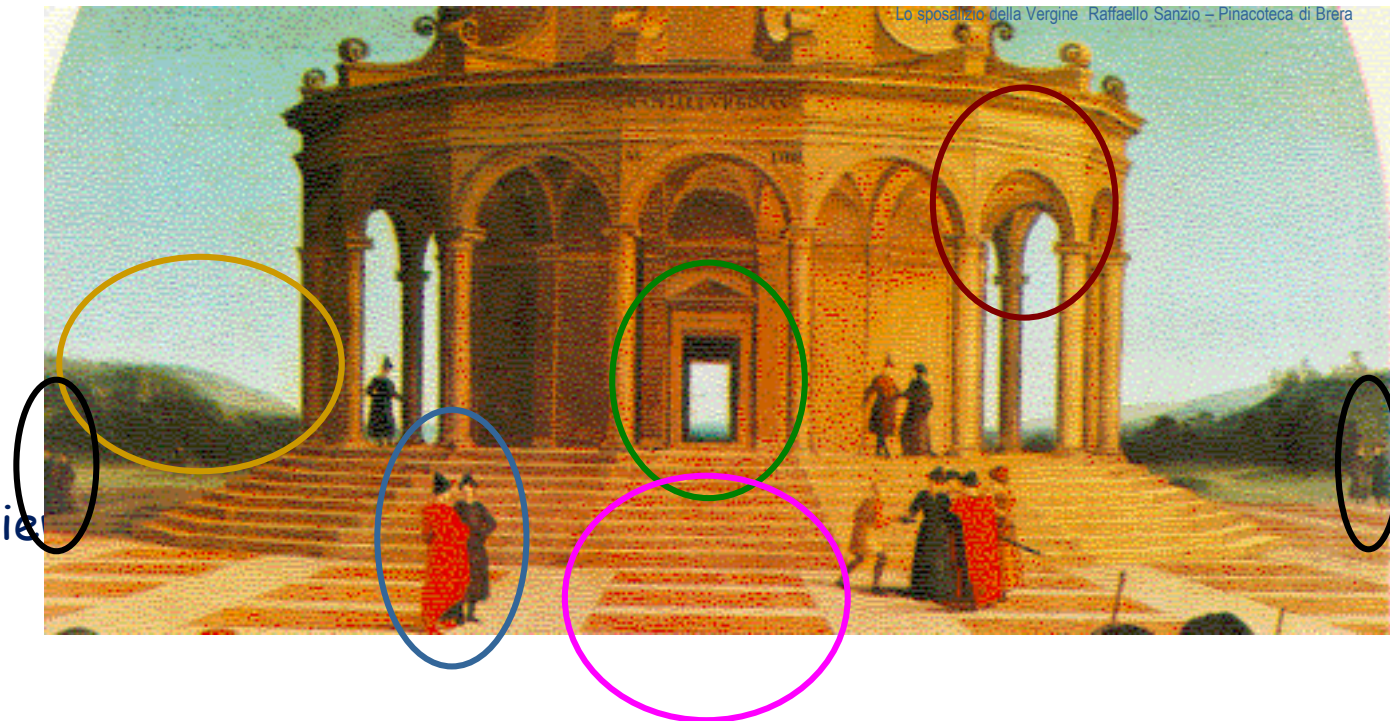
Perspective geometry

Depth from shading

Multi-presence

Depth from texture

Height in the field of view



Atmospheric perspective

- Based on the effect of air on the color and visual acuity of objects at various distances from the observer.
- Consequences:
 - Distant objects appear bluer
 - Distant objects have lower contrast.

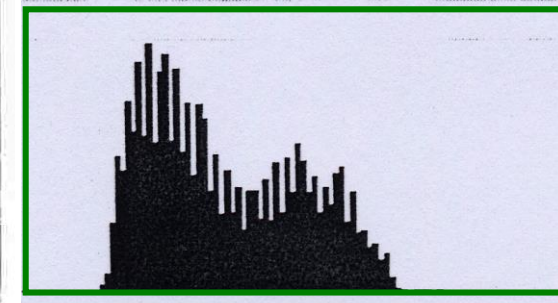
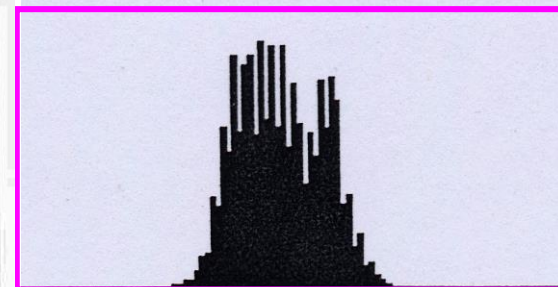
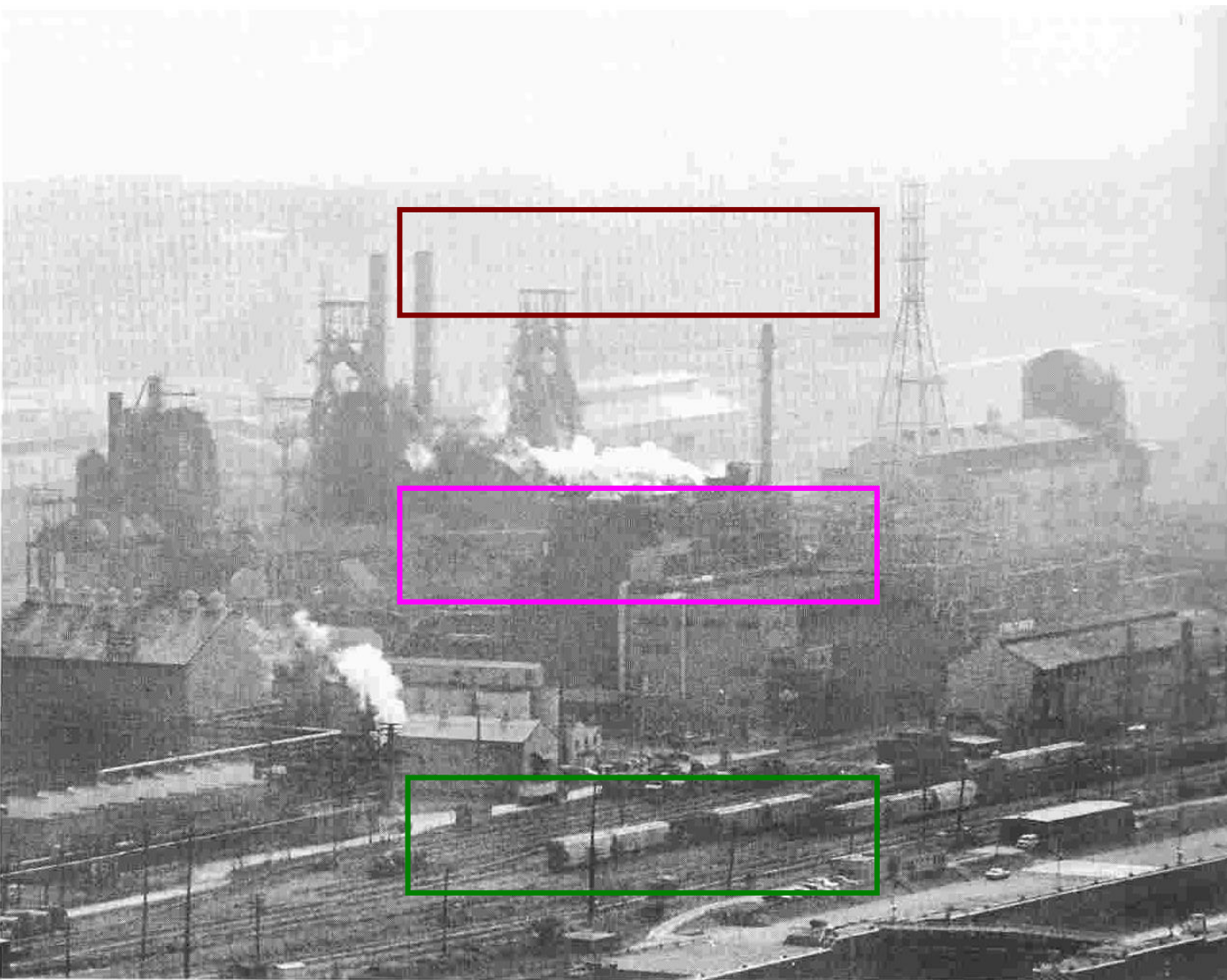


Atmospheric perspective

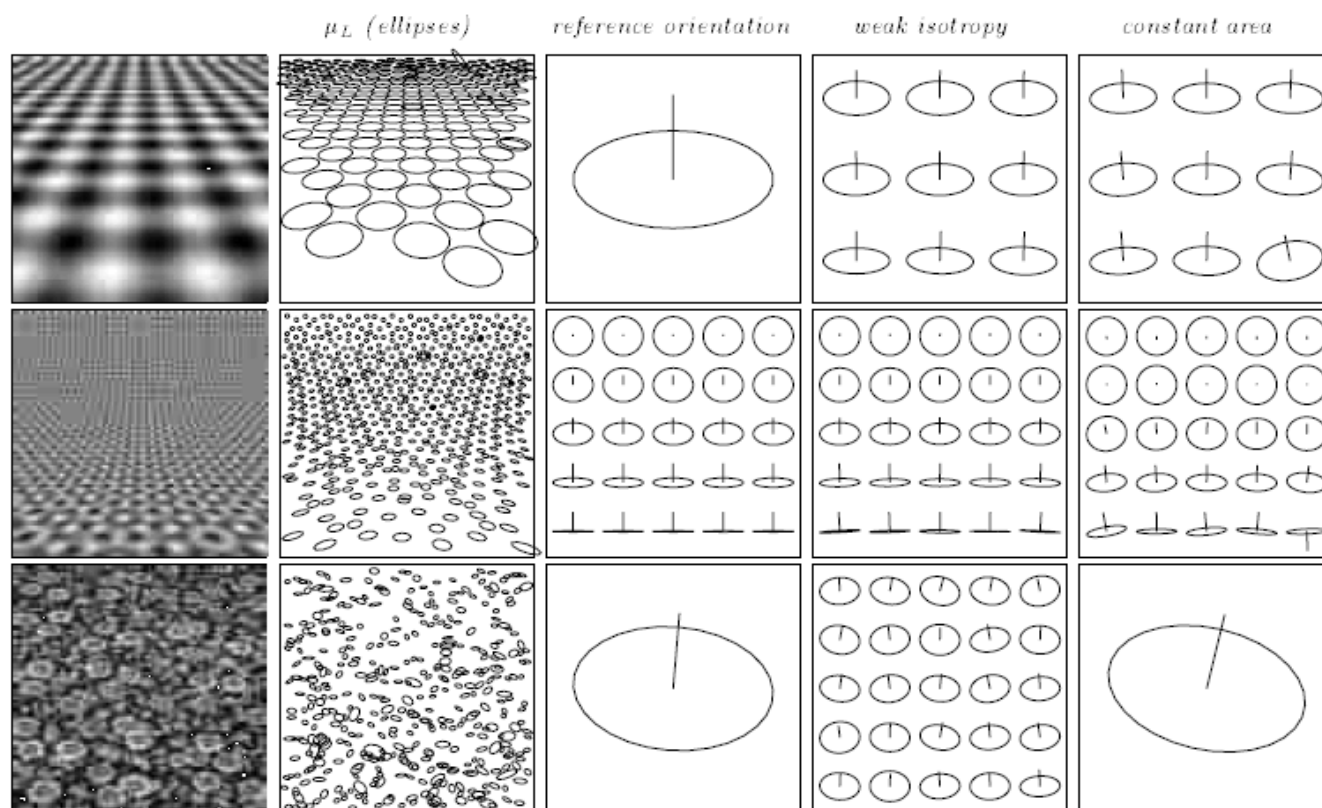


Claude Lorrain (artist) French, 1600 - 1682
Landscape with Ruins, Pastoral Figures, and Trees, 1643/1655

Histogram



Texture Gradient



Occlusion



Rene Magritte famous painting *Le Blanc-Seing* (literal translation: "The Blank Signature")

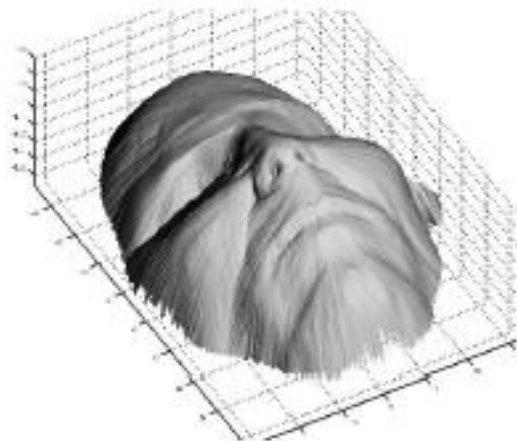
Shape from.... shading



a)



b)



c)

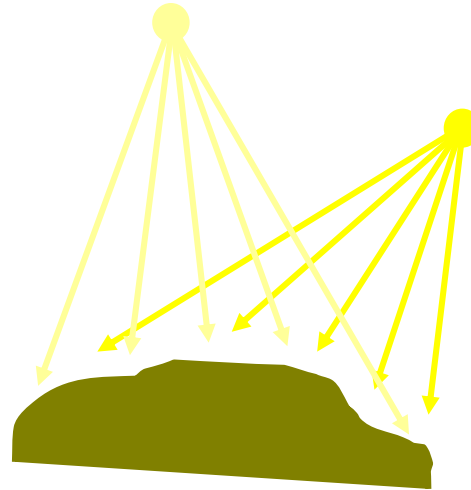


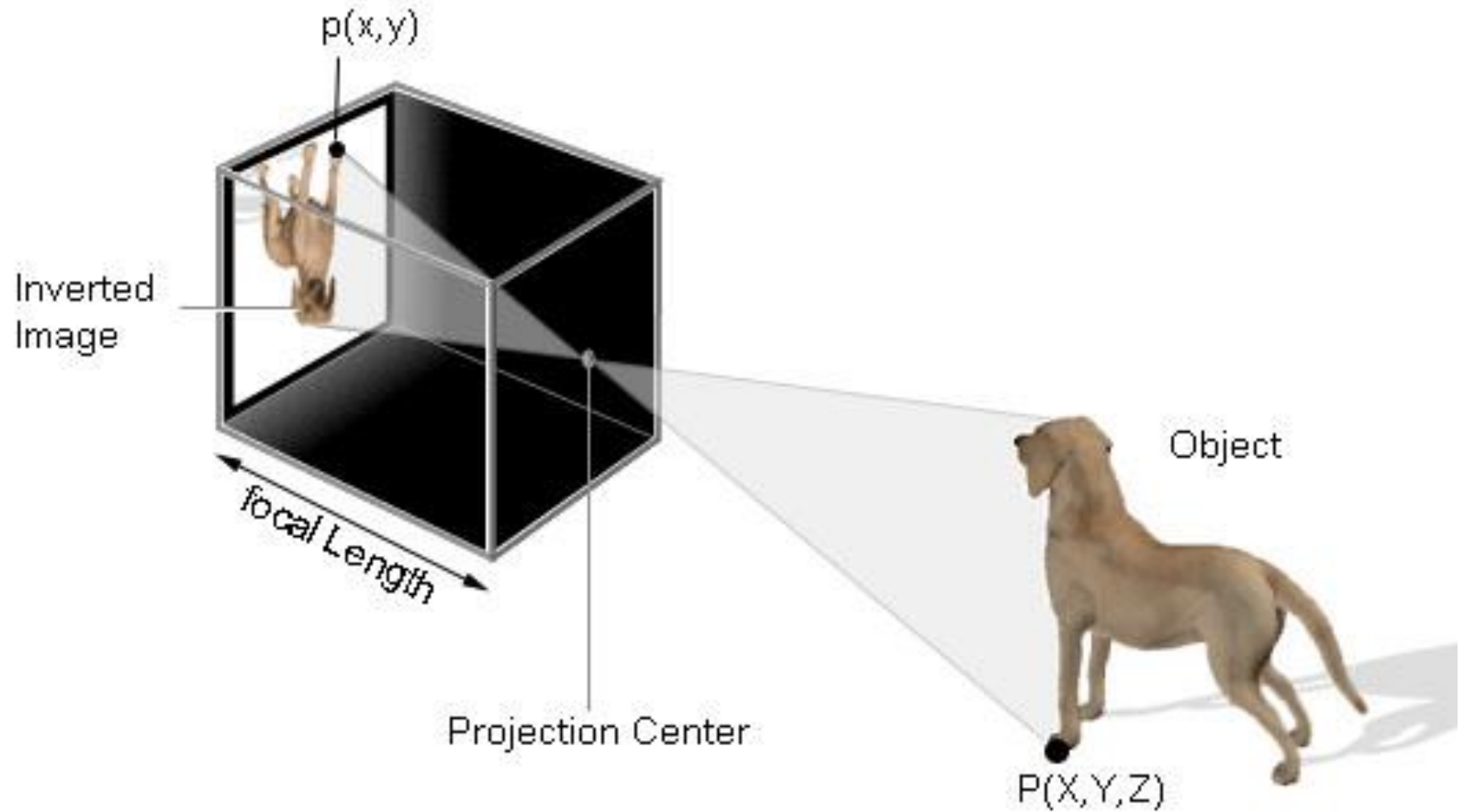
Figure from Prados & Faugeras 2006

Michelangelo 1528
20

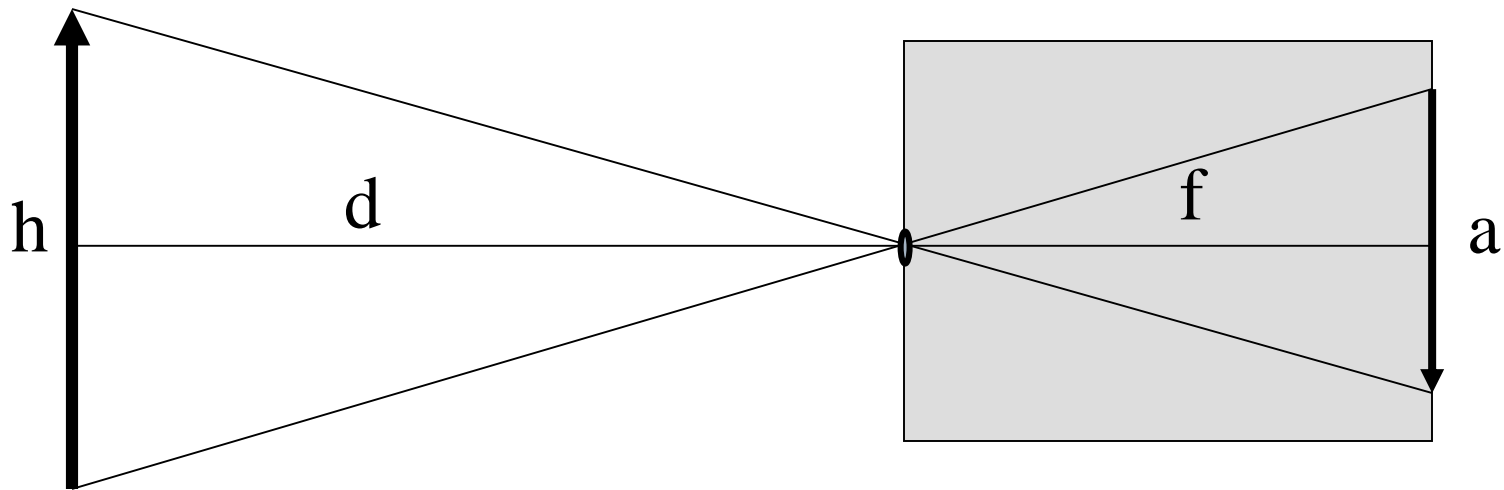
Shadows



Pinhole camera model

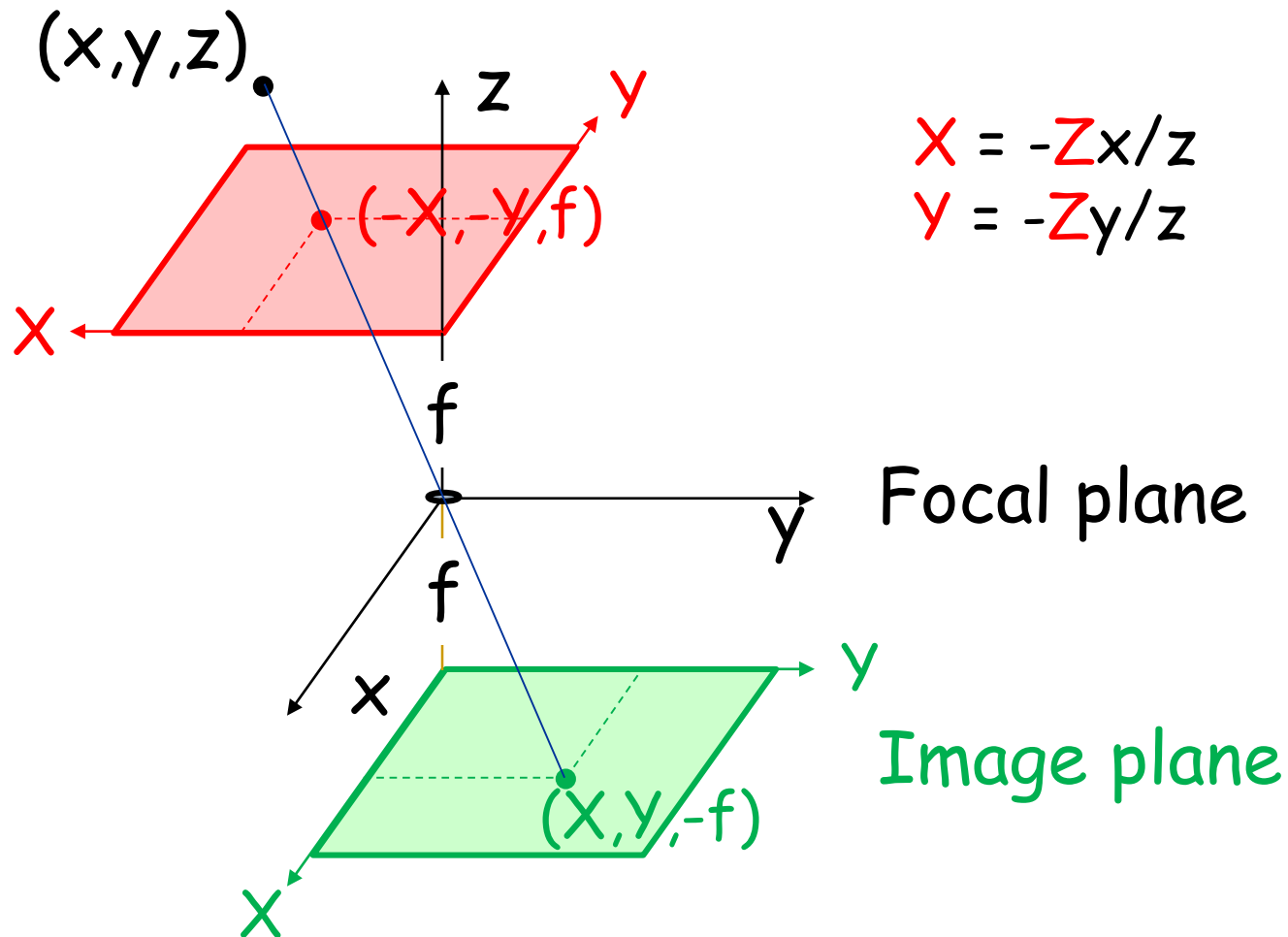


Pinhole camera model



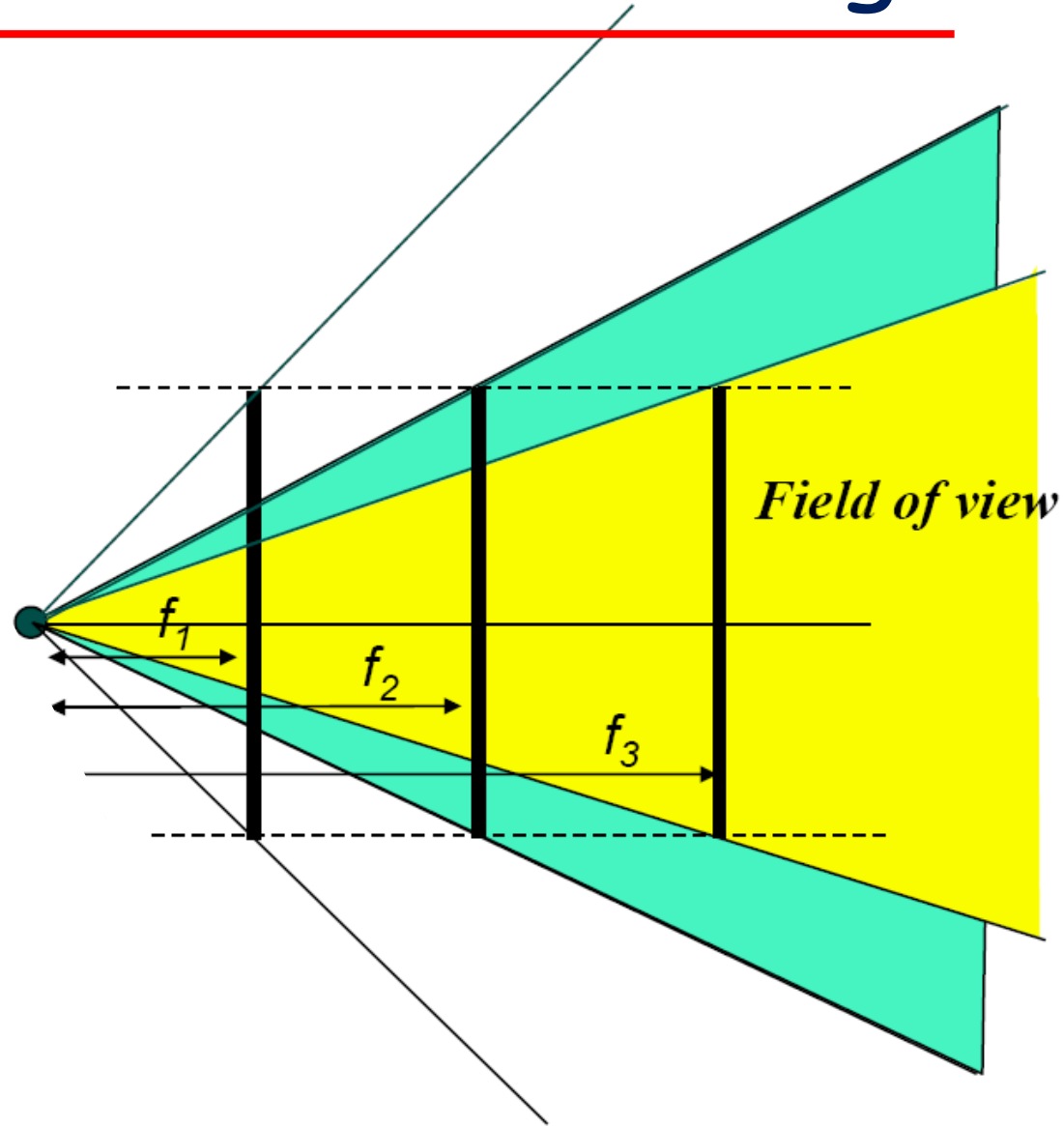
$$h/d = a/f$$

Geometry of the camera



Field of view depends on focal length

- As f gets smaller, image becomes more *wide angle*
 - more world points project onto the finite image plane
- As f gets larger, image becomes more *telescopic*
 - smaller part of the world projects onto the finite image plane



Field of view: portion of 3d space seen by the camera



28 mm lens, $65.5^\circ \times 46.4^\circ$



50 mm lens, $39.6^\circ \times 27.0^\circ$



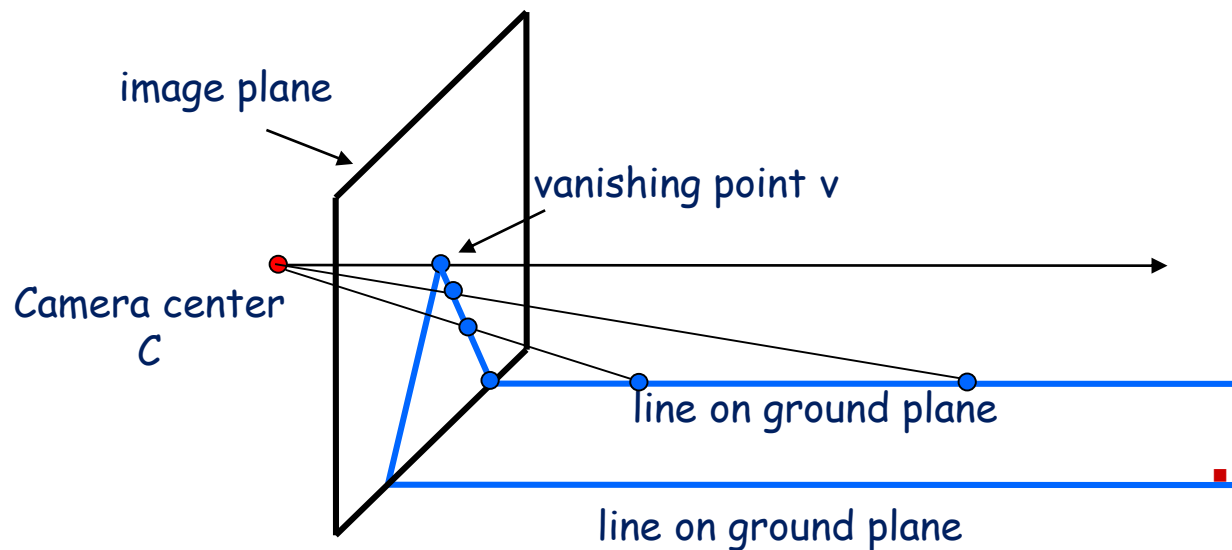
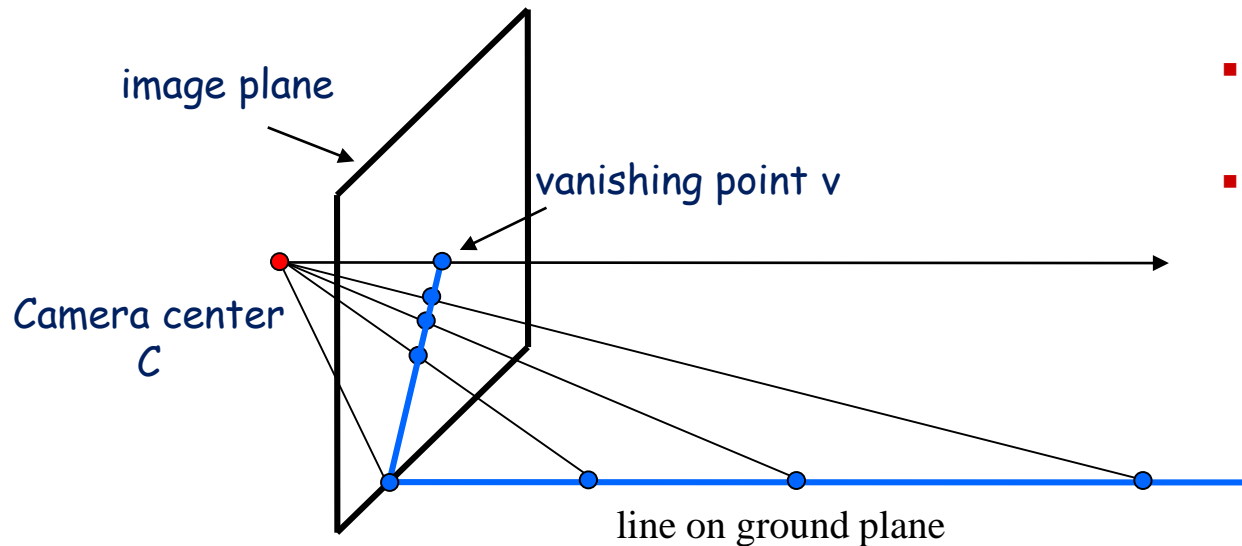
70 mm lens, $28.9^\circ \times 19.5^\circ$



210 mm lens, $9.8^\circ \times 6.5^\circ$

Vanishing points: projection of a point at infinity

- Parallel lines in the scene intersect in the image
- Converge in image on horizon line



■ An image may have more than one vanishing point

Perspective effects

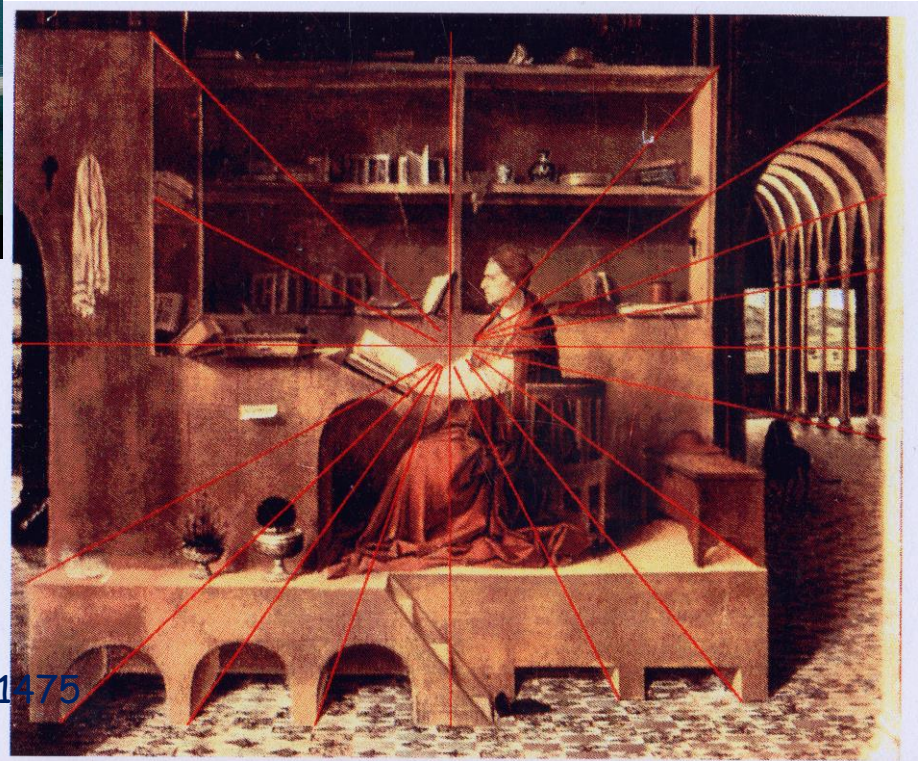


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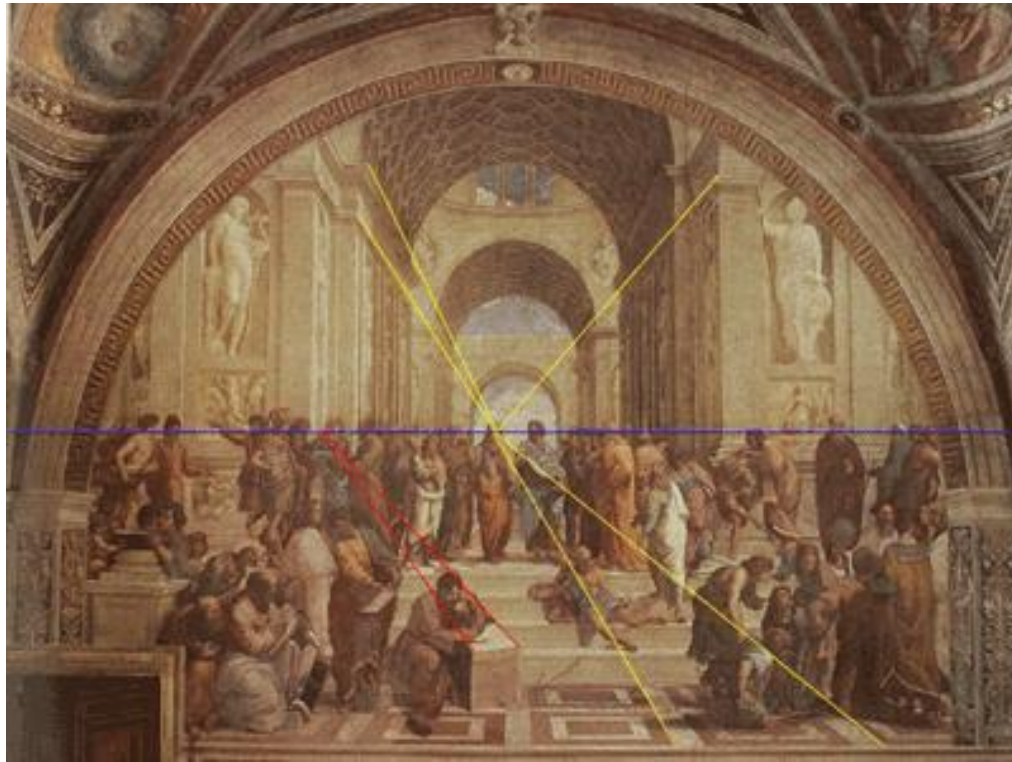
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Image credit: S. Seitz

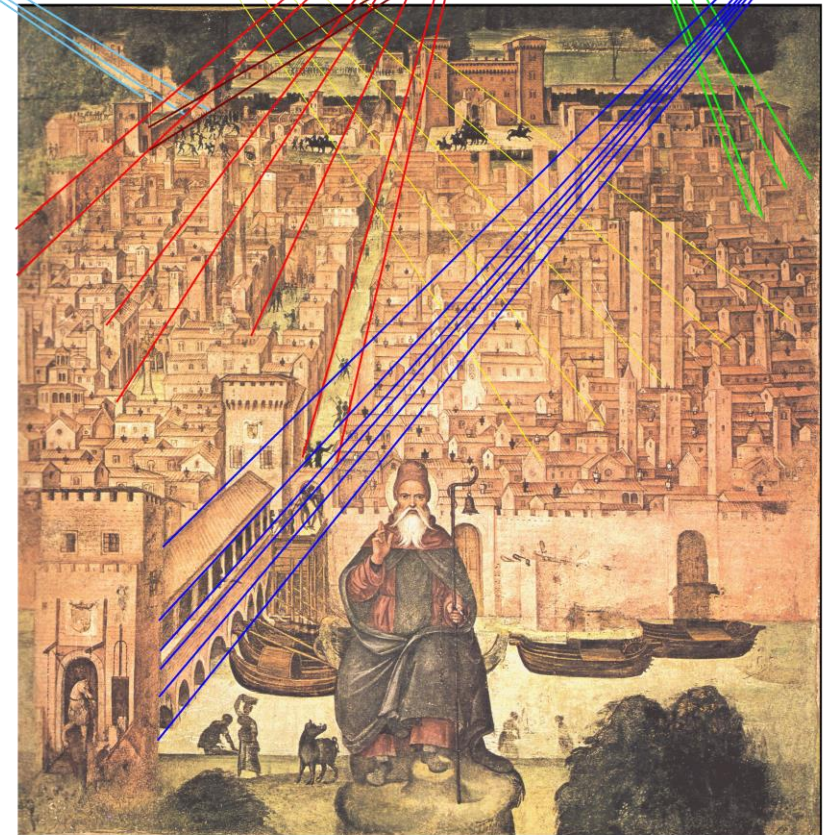
San Girolamo nello studio
Antonello da Messina, 1474-1475
London, National Gallery



Perspective effect

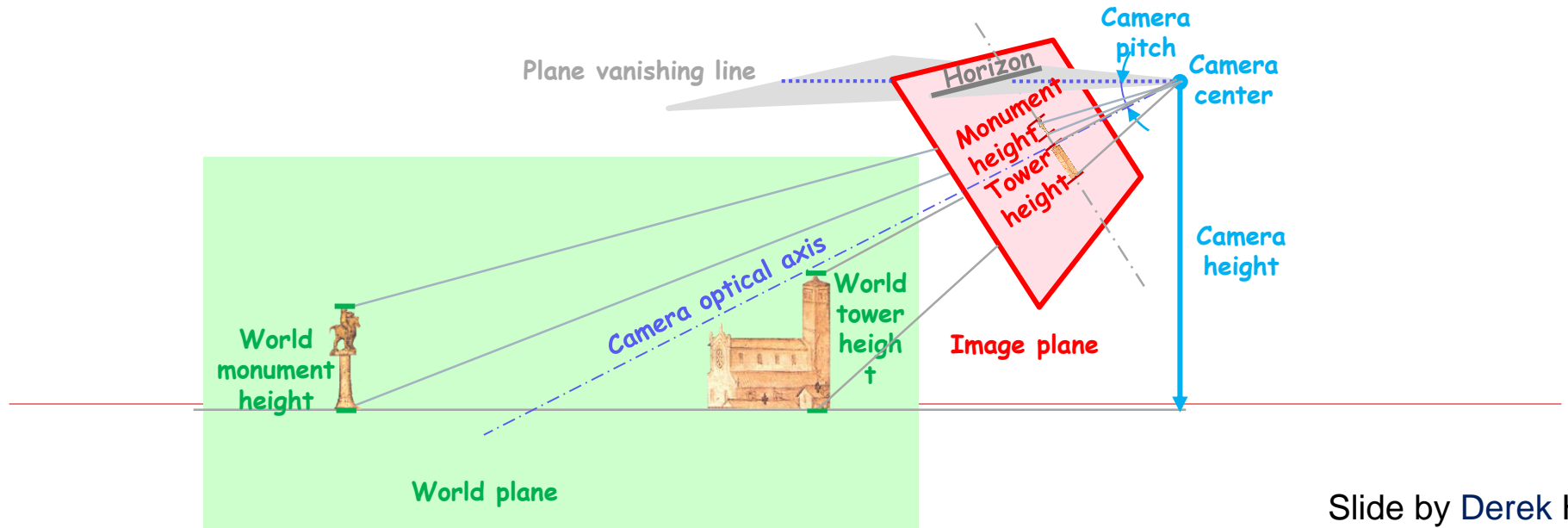
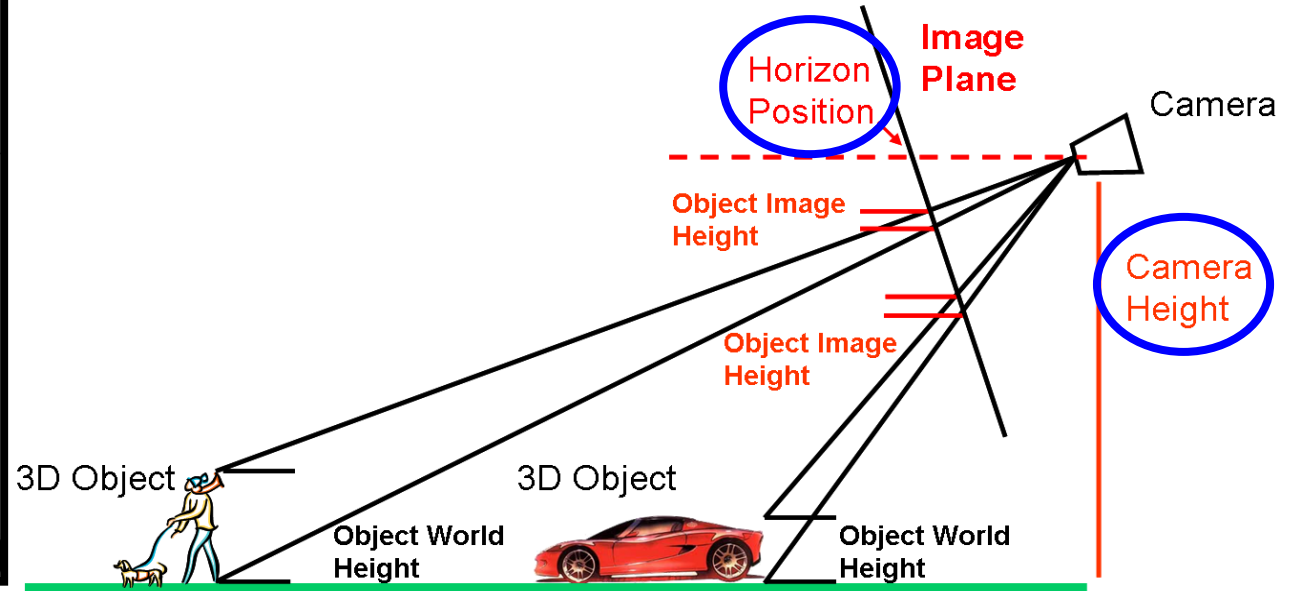
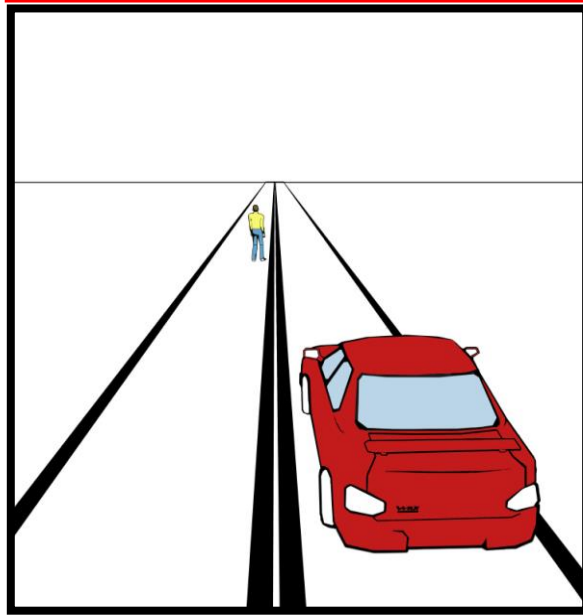


The School of Athens by Raffaello Sanzio
which dates from 1508-1511



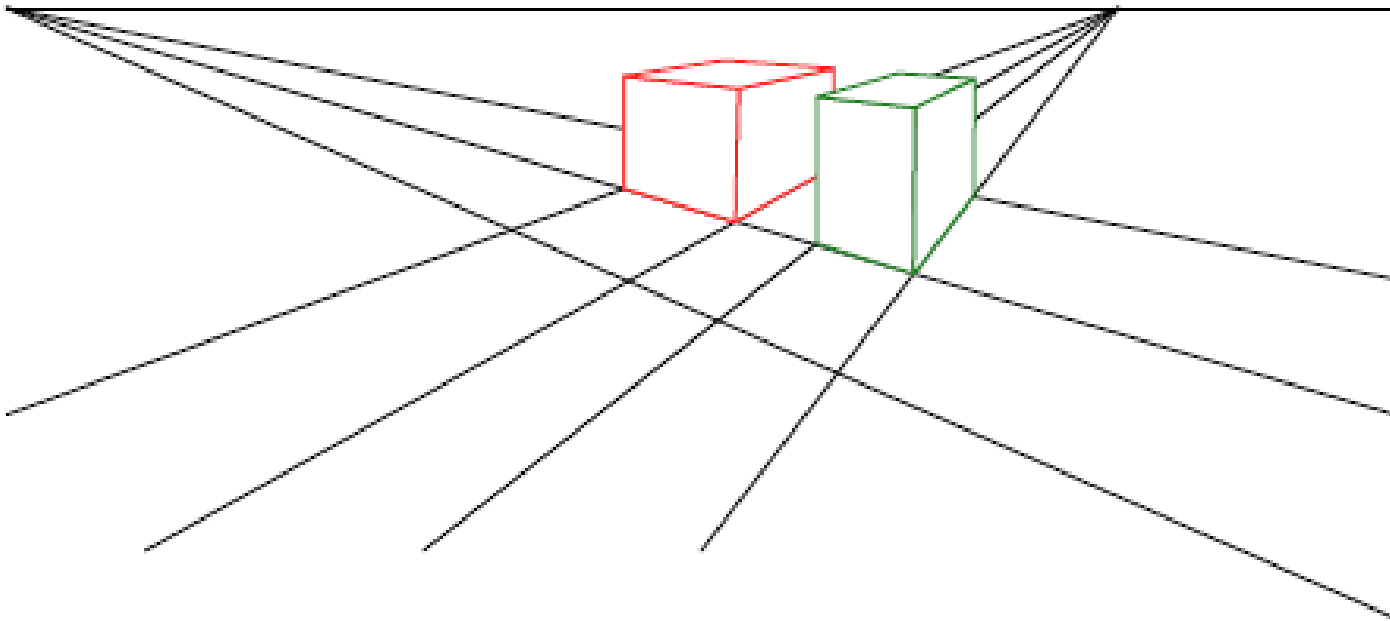
City of Pavia attributed to Bernardino Lanzani
which dates from 1522

Object Size in the Image

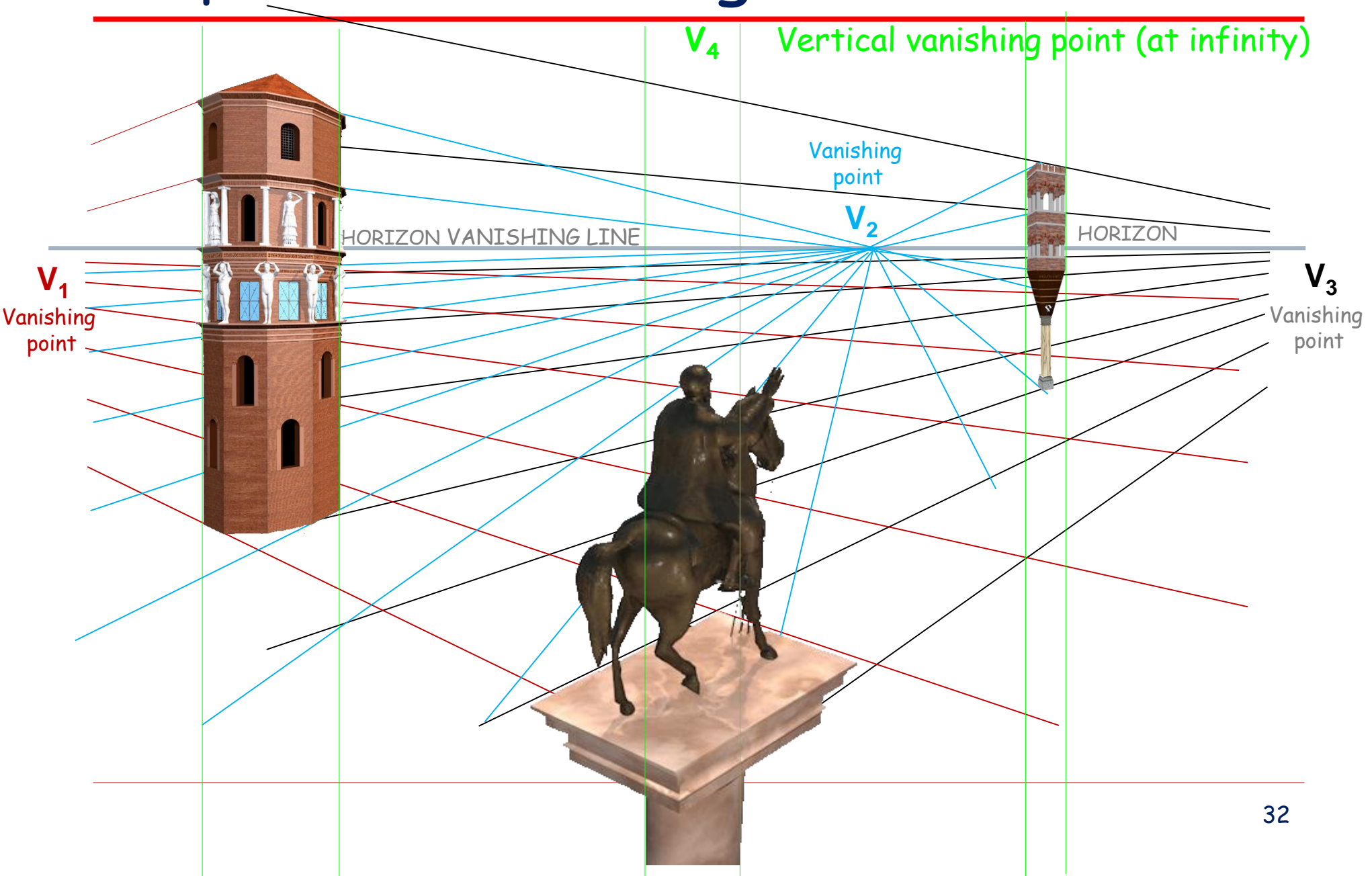


Vanishing points

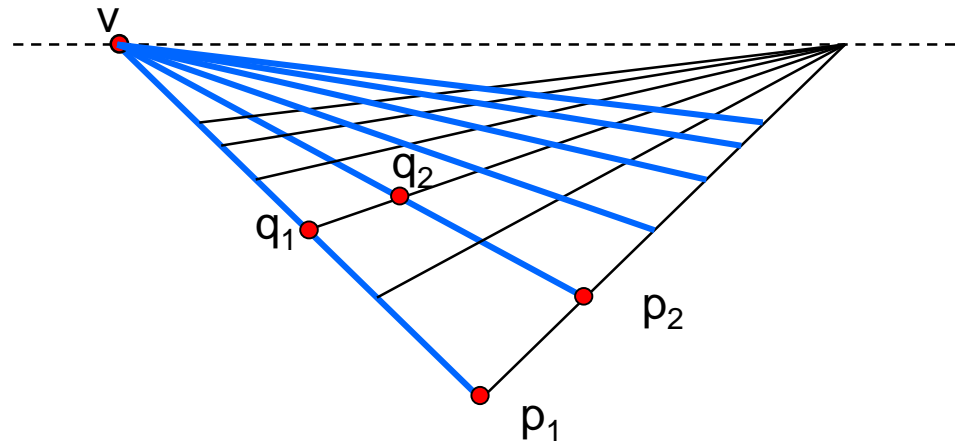
- Each set of parallel lines (=direction) meets at a different point
 - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
 - The line is called the *horizon* for that plane



Perspective cues heights and locations

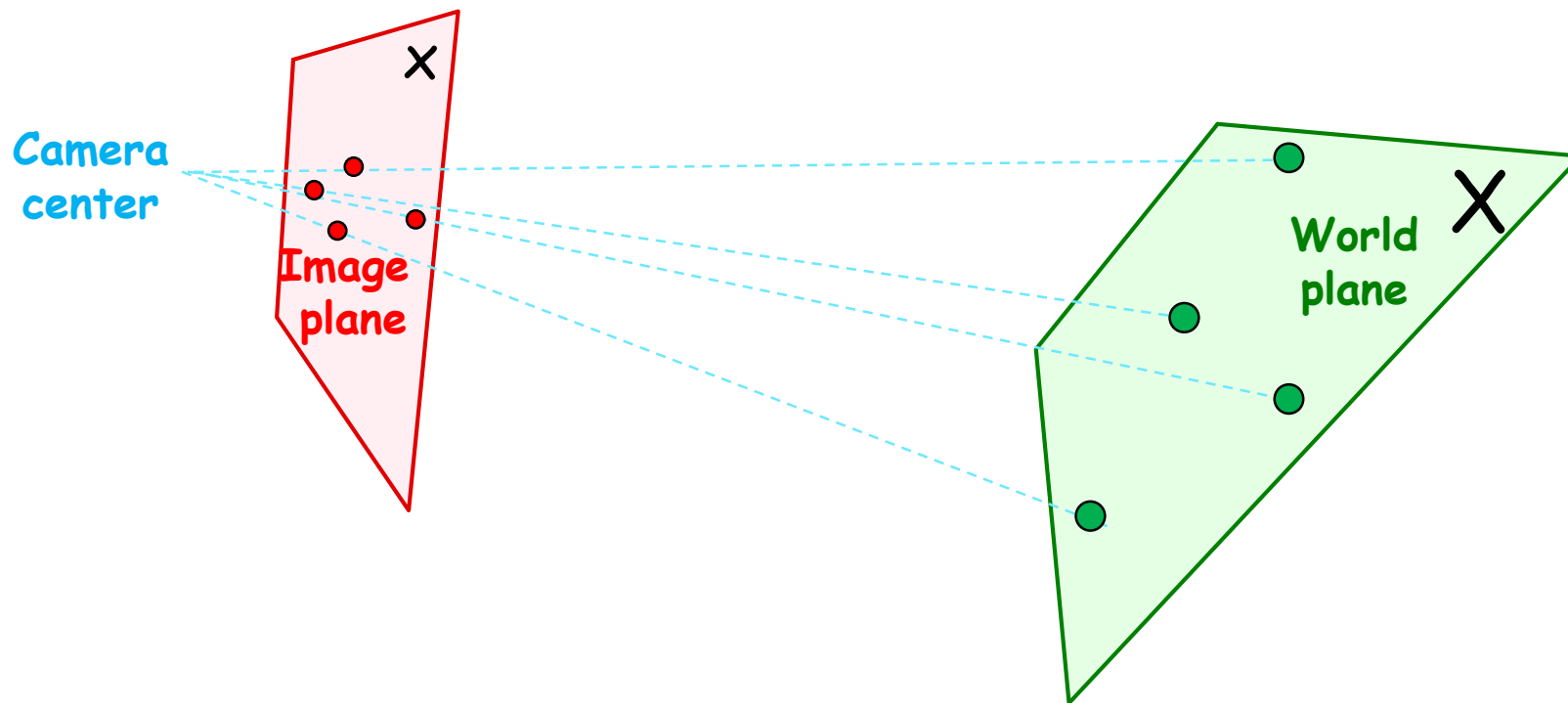


Computing vanishing points (from lines)



- Intersect p_1q_1 with p_2q_2
- Better to use more than two lines and compute the "closest" point of intersection (Least squares version)
- See notes by [Bob Collins](http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt) for one good way of doing this:
<http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt>

Homography : a plane to plane projective transformation

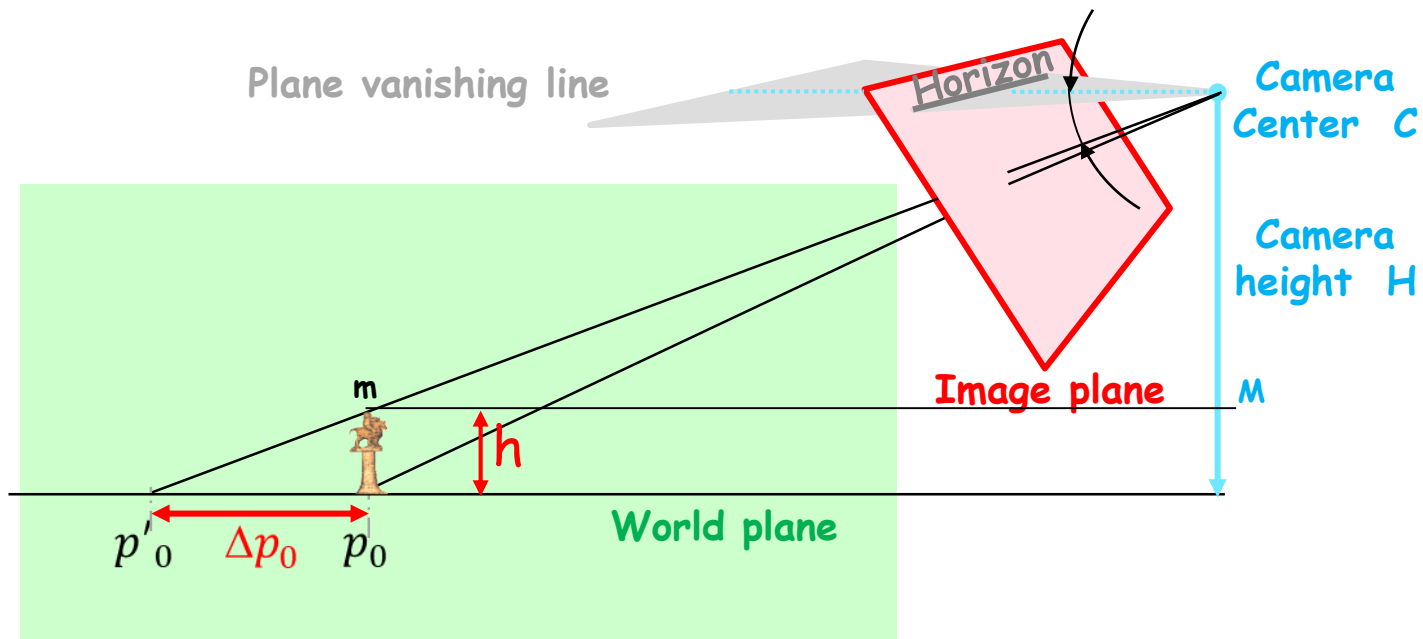


Homography matrix: $X = Hx$

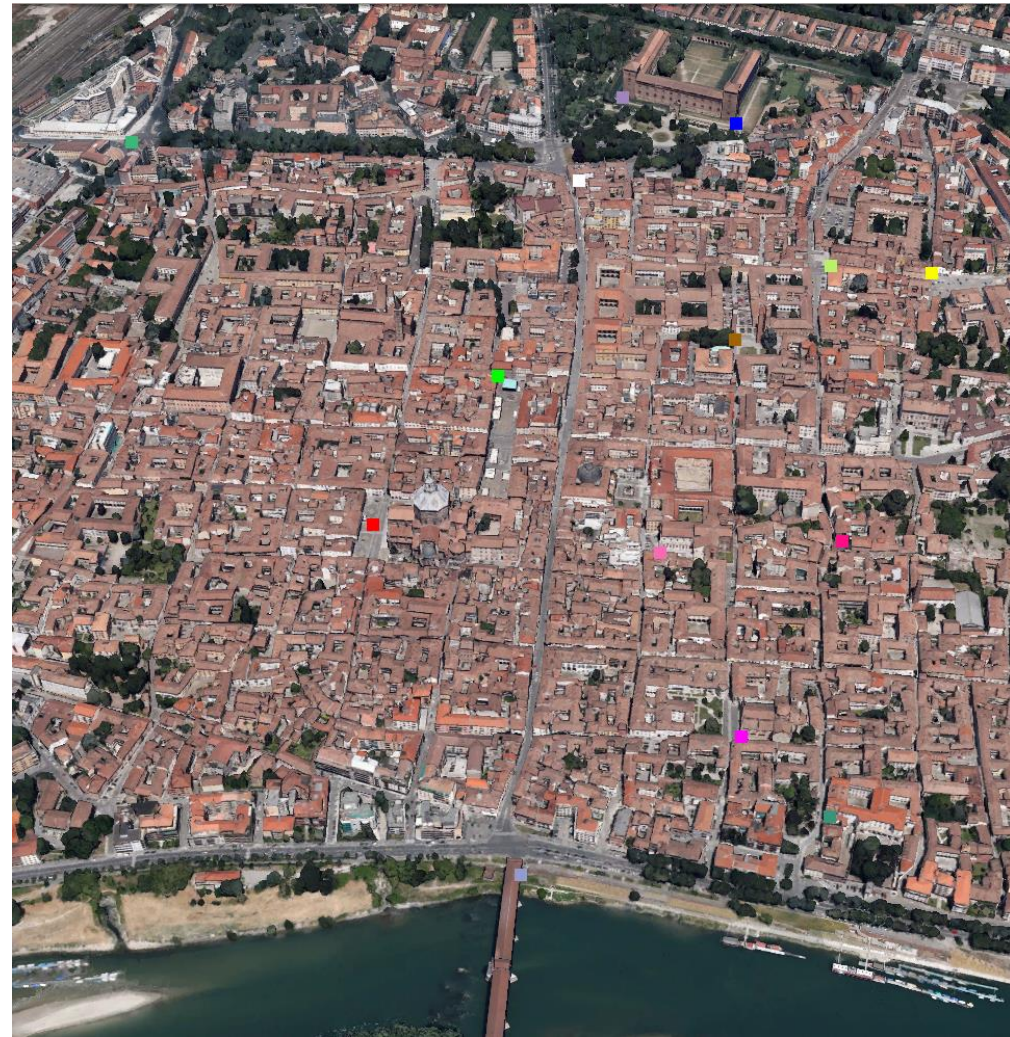
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$x = \frac{u}{w}, y = \frac{v}{w}$$

Object Size in the Image



Tie points



Homography: Google Earth map on the fresco



Homography: the fresco on Google Earth map

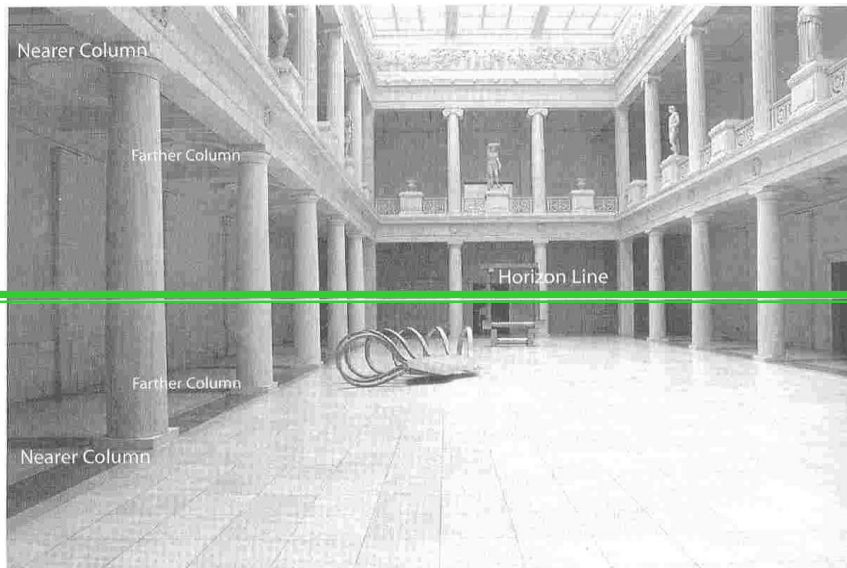


Distance from the horizon line



Distance from the horizon line

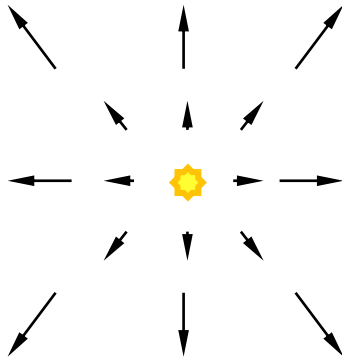
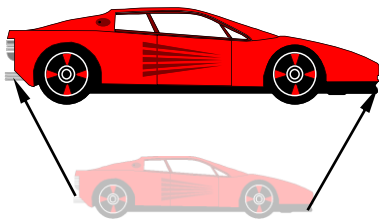
- Based on the tendency of objects to appear **nearer** the horizon line with **greater** distance to the horizon.
- Objects **above** the horizon that appear **lower** in the field of view are seen as being **further** away.
- Objects **below** the horizon that appear **higher** in the field of view are seen as being **further** away.



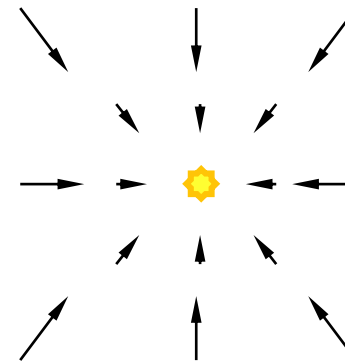
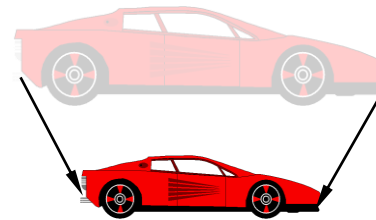
- Objects approach the horizon line with greater distance from the viewer.
- The base of a nearer column will appear lower against its background floor and further from the horizon line.
- Conversely, the base of a more distant column will appear higher against the same floor, and thus nearer to the horizon line.

Distance from egomotion

Focus of **expansion**



Focus of **contraction**



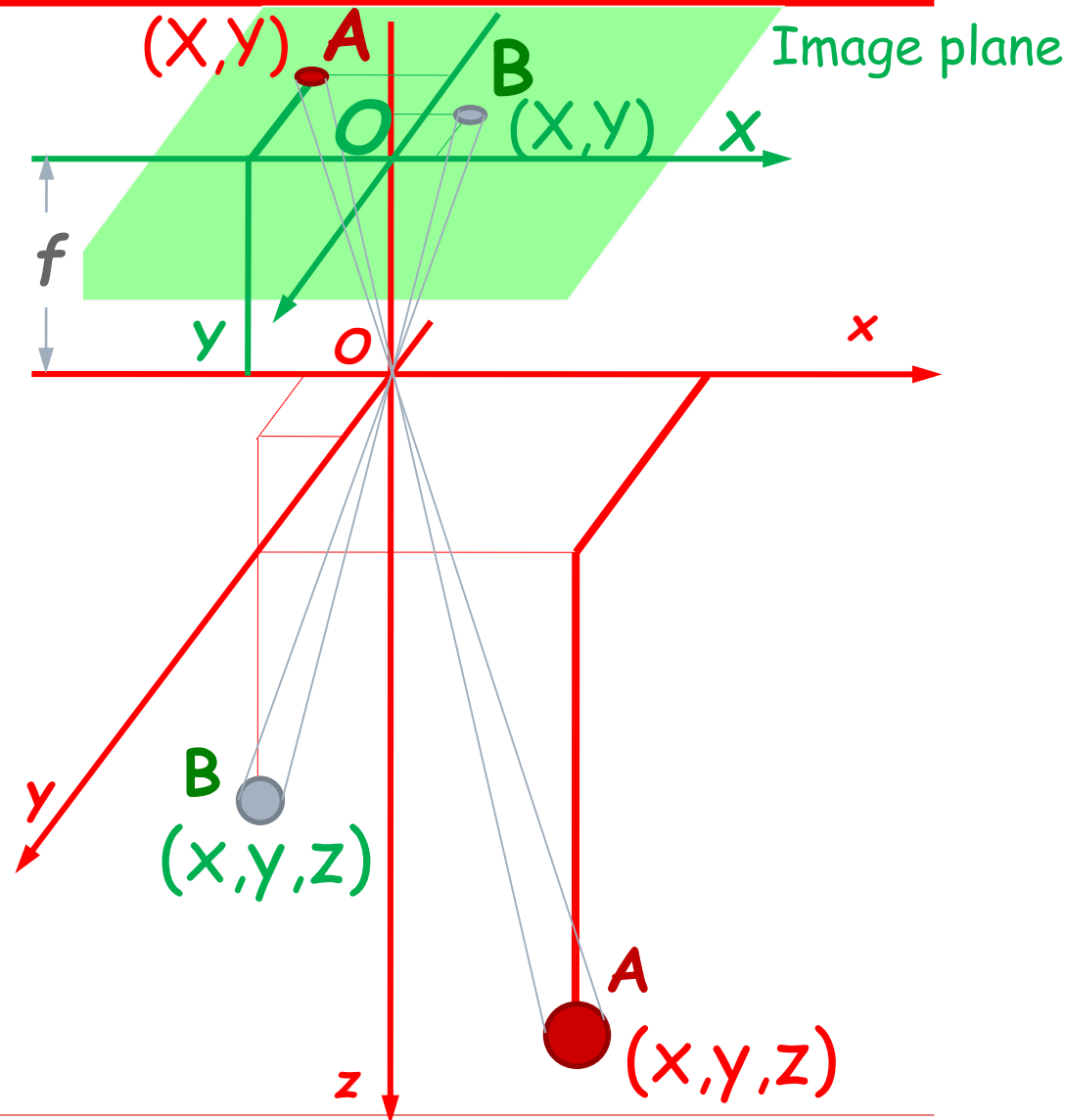
Distance from egomotion

$$\frac{Y}{y} = -\frac{f}{z}$$

$$\frac{\partial Y}{\partial z} = \frac{yf}{z^2} = -\frac{Y}{z}$$

Impact time estimation

$$z = -\frac{Y \partial z}{\partial Y}$$



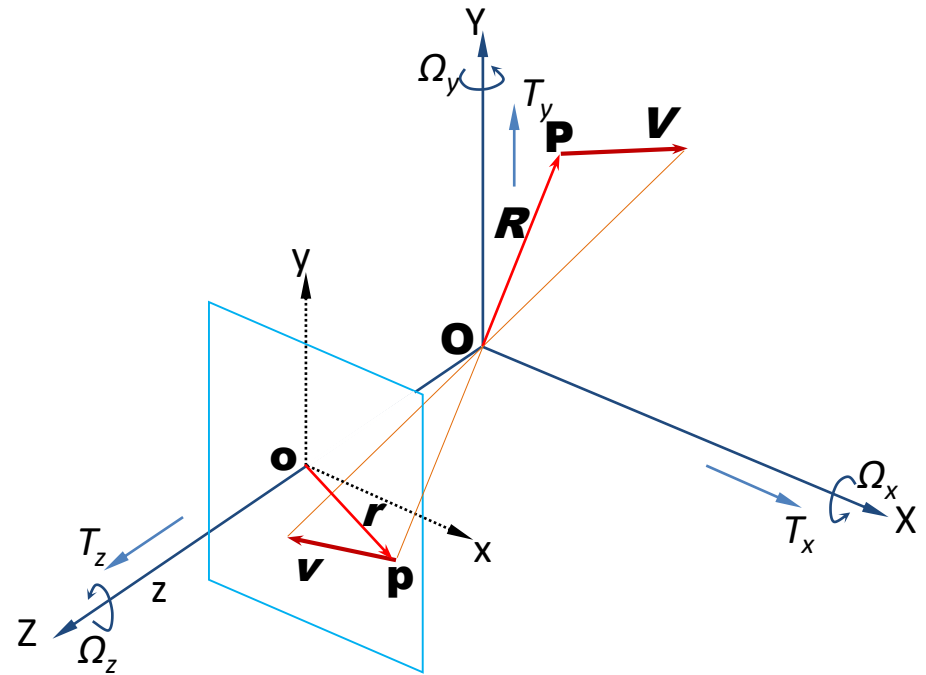
Camera and motion models

- The egomotion makes all still objects in the scene to verify the same motion model defined by **three translations T** and **three rotations Ω** . Conversely, mobile obstacles pop out as not resorting to the former dominating model.
- Under such assumptions, the following classical equations hold:

$$u_t = \frac{-fT_X + xT_Z}{Z}, \quad u_r = \frac{-xy}{f}\Omega_X - \left(\frac{-x^2}{f} + 1\right)\Omega_Y + y\Omega_Z$$

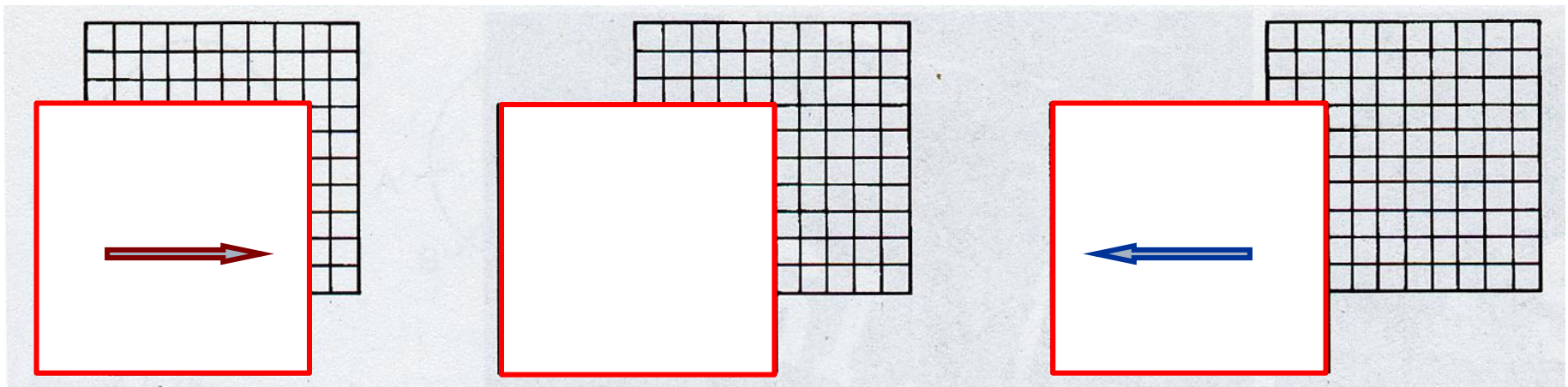
$$v_t = \frac{-fT_Y + yT_Z}{Z}, \quad v_r = \frac{-xy}{f}\Omega_Y - \left(\frac{-y^2}{f} + 1\right)\Omega_X + x\Omega_Z$$

- where $\mathbf{w} = [u, v]^T = [u_t + u_r, v_t + v_r]^T$ stands for the 2-D velocity vector of the pixel under the focal length f .



Motion occlusion and egomotion

Deletion and accretion occur when an observer moves in a **direction not perpendicular** to two surfaces that are at **different depths**. If an observer perceives the two surfaces as in the center and then moves to the left, deletion occurs so that the front object covers more that the back one, as shown on the left. Vice versa for the movement in the opposite direction as shown on the right

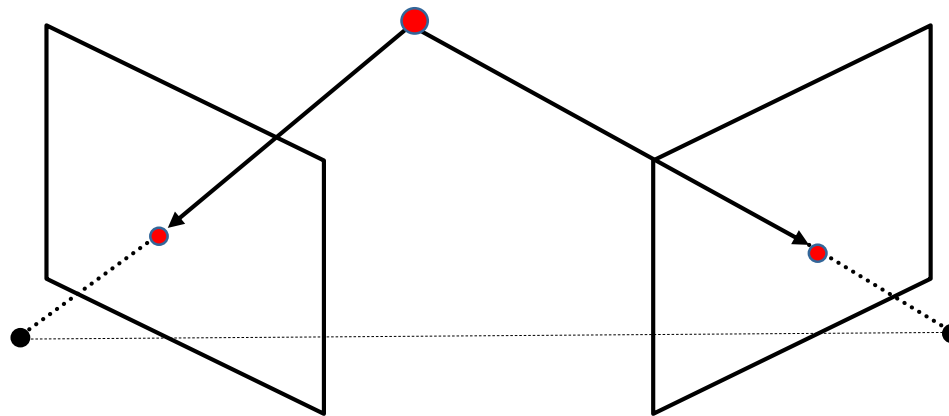


Deletion

Initiale position

Accretion

Stereo: Epipolar geometry



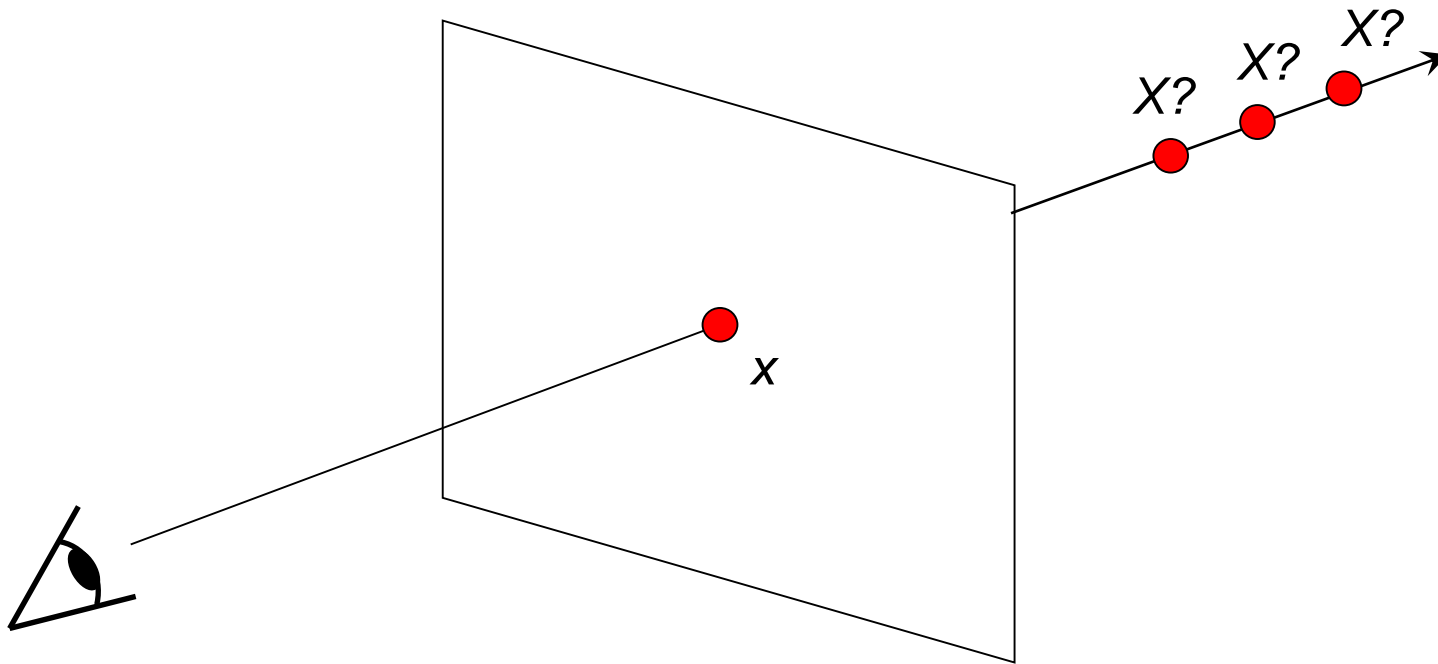
Why multiple views?

- Structure and depth are inherently ambiguous from single views.

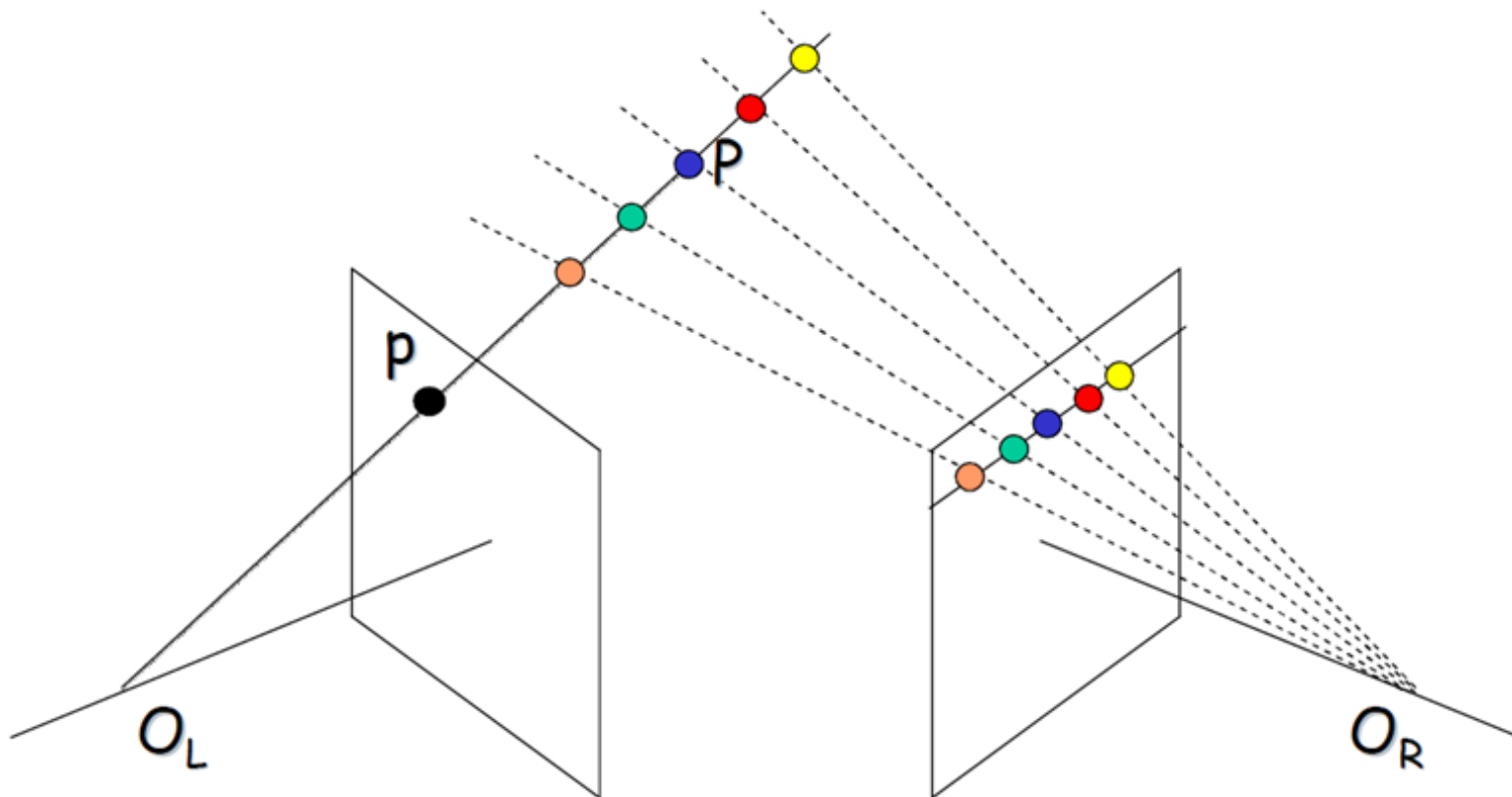


Our goal: Recovery of 3D structure

- Recovery of structure from one image is inherently ambiguous



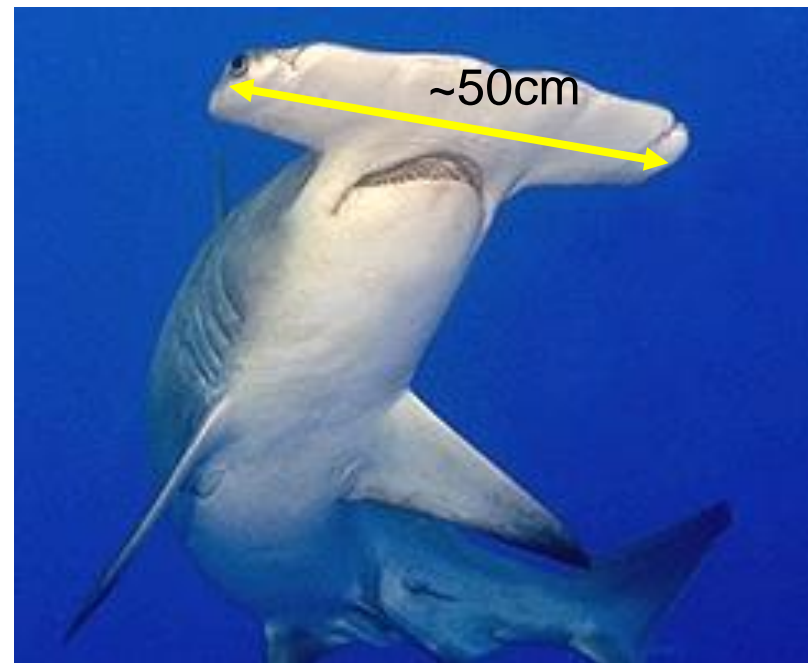
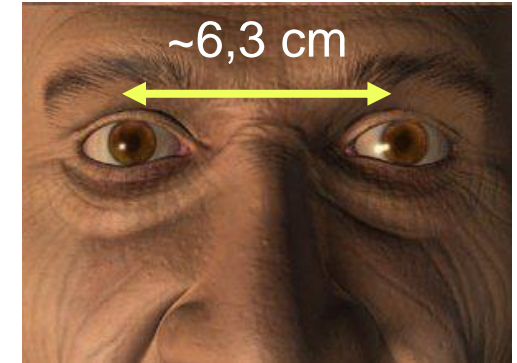
Why stereo vision?



- A second camera can resolve the ambiguity, enabling measurement via triangulation
-

Stereo vision

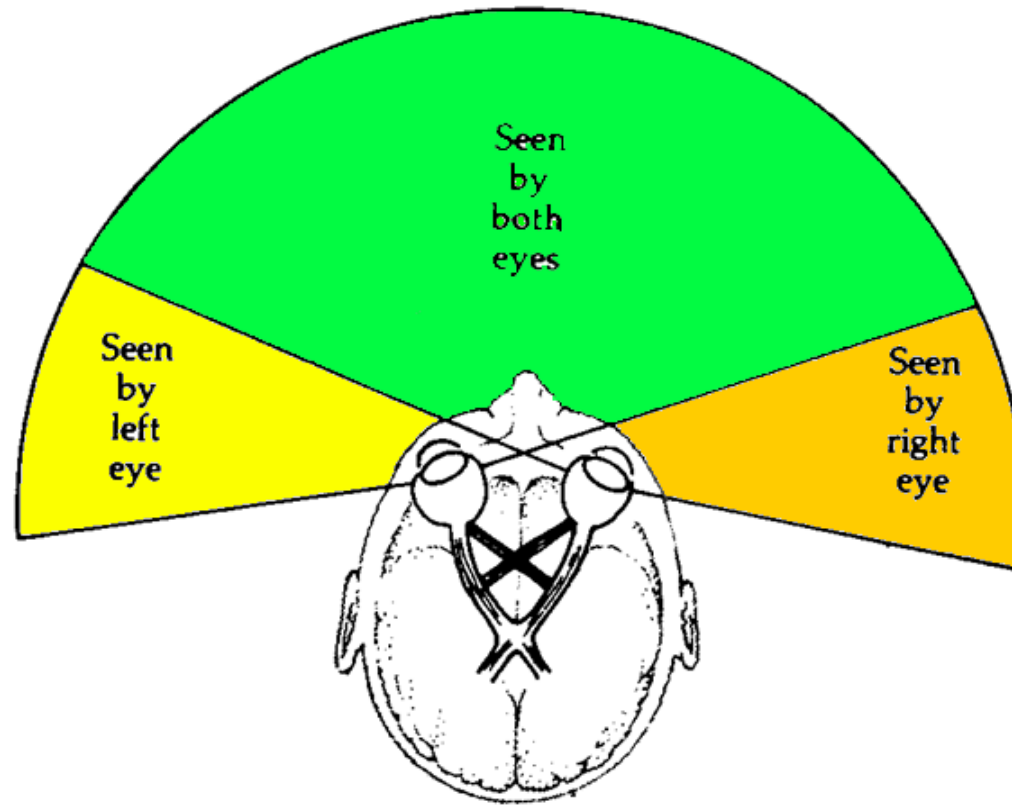
- After 10 meters (30 feet) disparity is quite small and depth from stereo is unreliable...



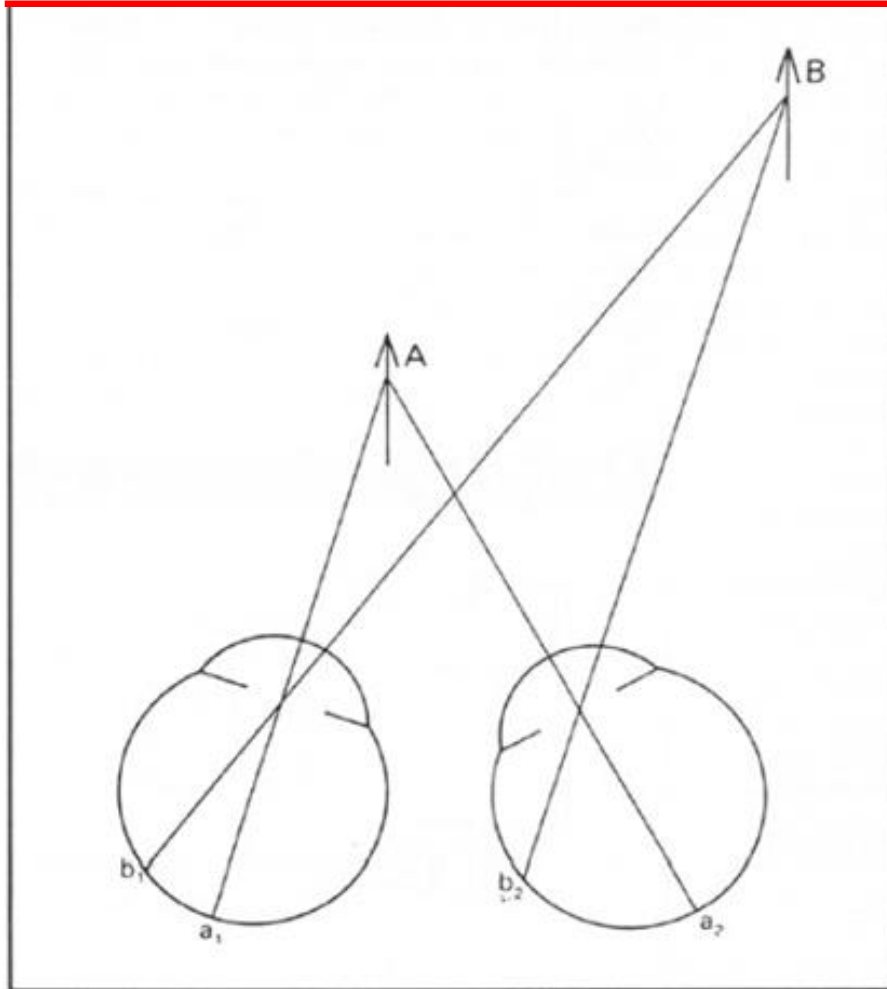
Visual Field

Monocular Visual Field: 160 deg (w) X 135 deg (h)

Binocular Visual Field: 200 deg (w) X 135 deg (h)



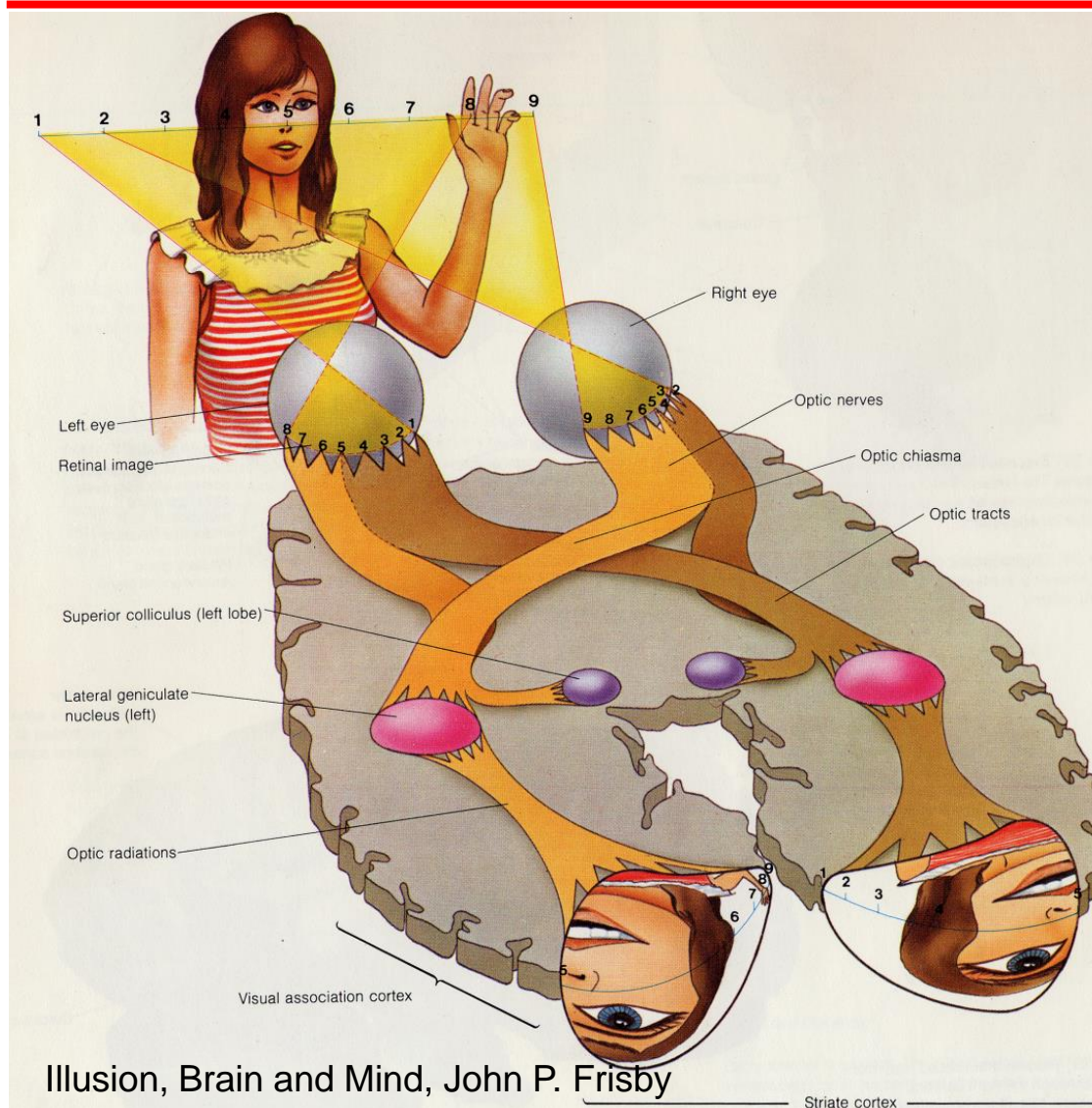
Human stereopsis: disparity



- Human eyes **fixate** on point **A** in space - rotate so that corresponding images form in centers of fovea.
- **Disparity** occurs when eyes fixate on one object; others (e.g. B) appear at different visual angles

From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Schema of the two human visual pathways



Illusion, Brain and Mind, John P. Frisby

BRAIN AND VISUAL PERCEPTION



The Story of a

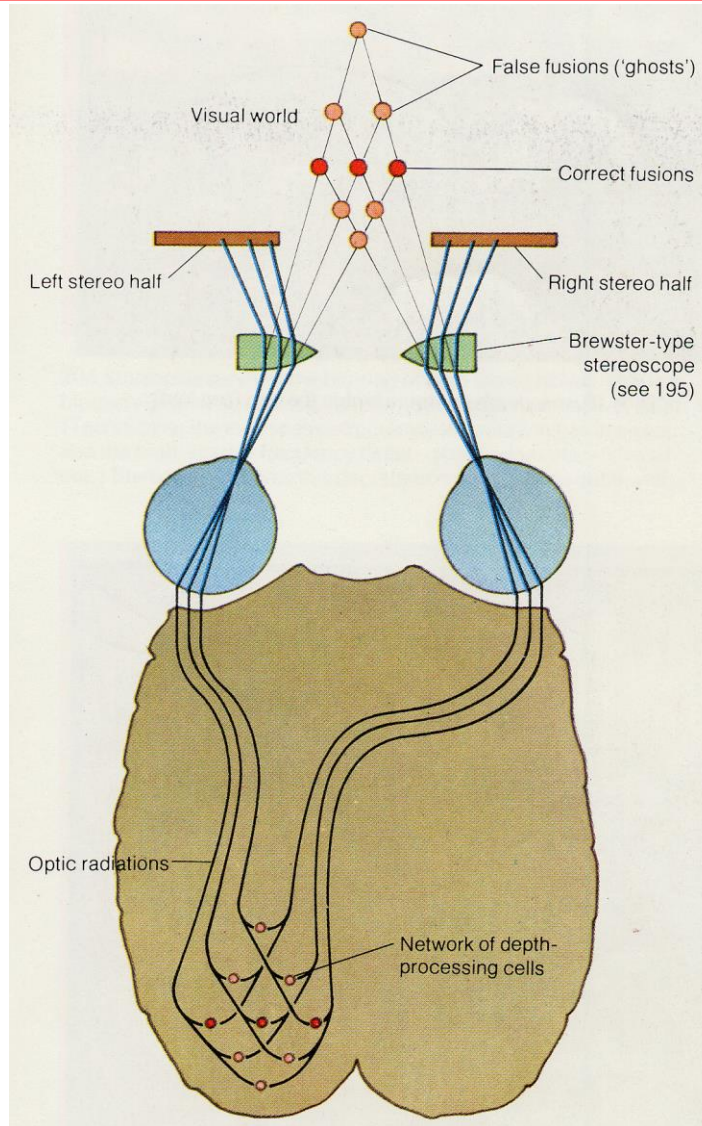


25-Year Collaboration

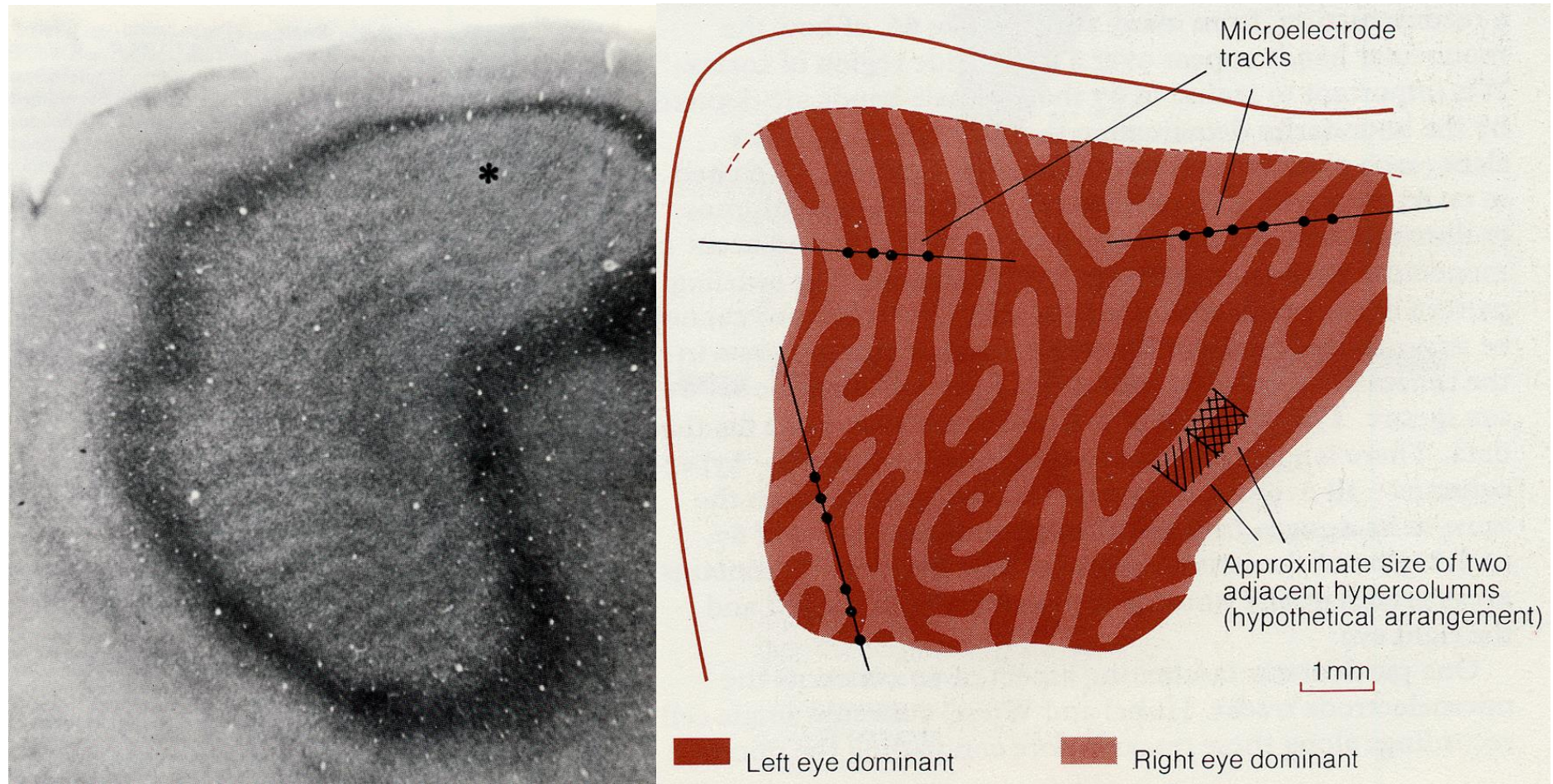


DAVID H. HUBEL • TORSTEN N. WIESEL

The problem of global stereopsis

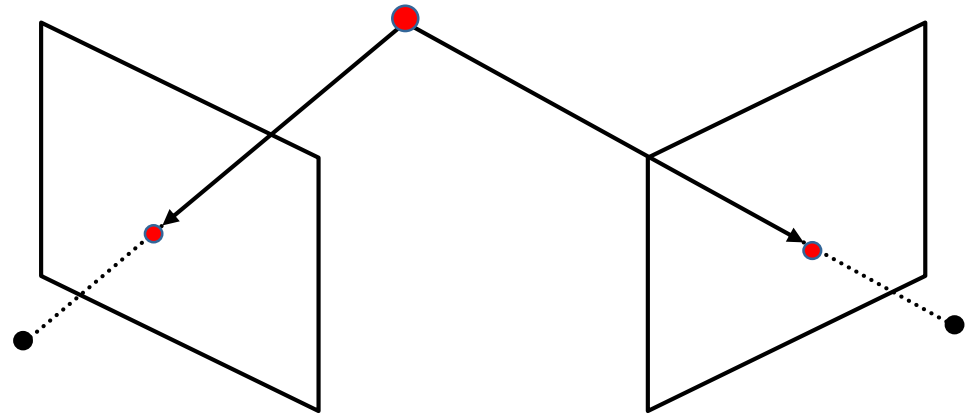
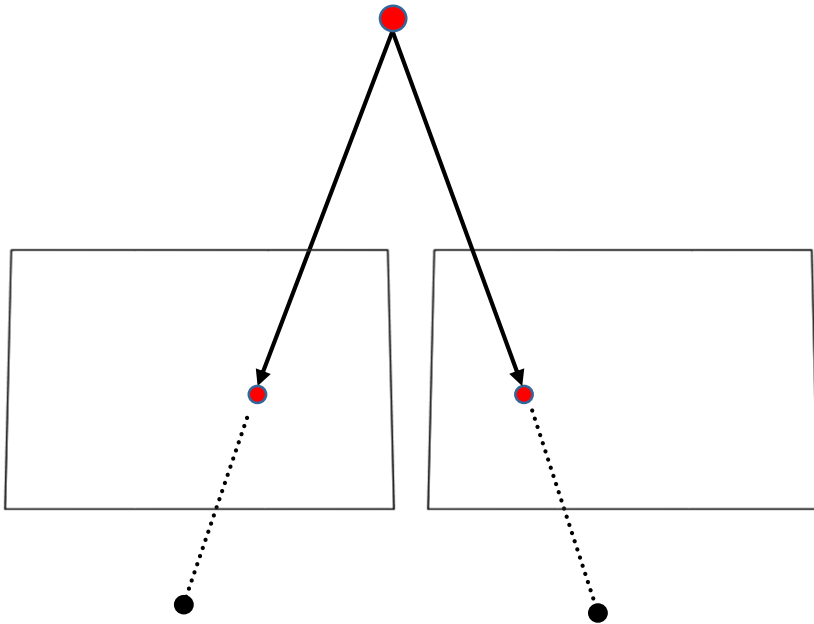


Section of striate cortex: schematic diagram of dominant band cells

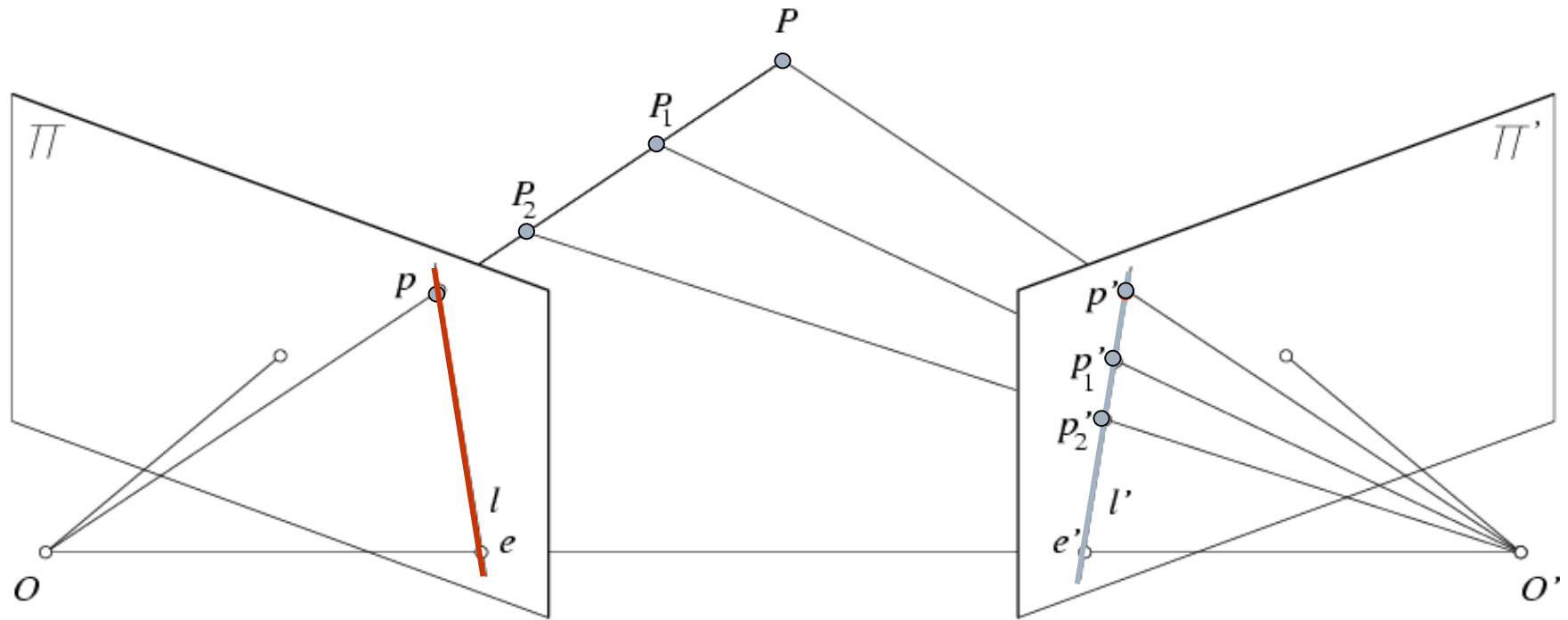


General case, with calibrated cameras

- The two cameras need not have parallel optical axes.



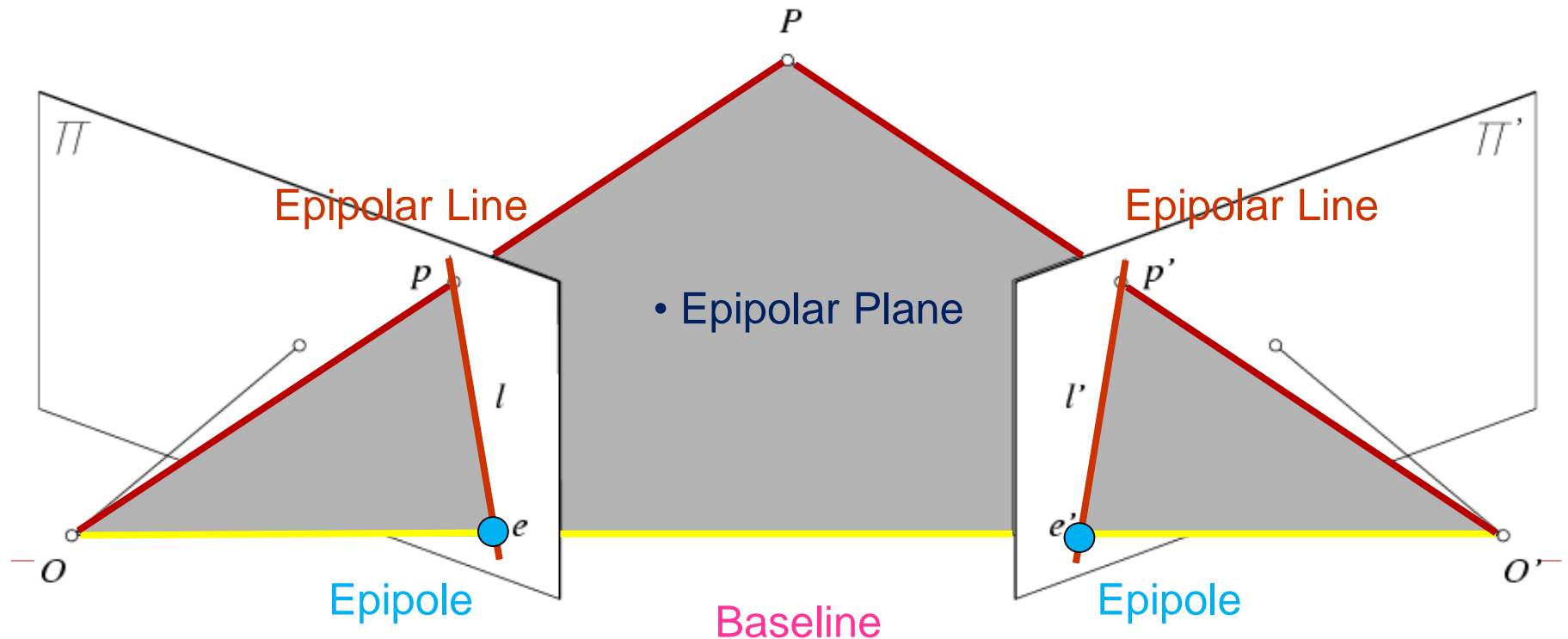
Epipolar constraint



- Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view.
 - It must be on the line carved out by a plane connecting the world point and optical centers.
-

Epipolar geometry: terms

- **Baseline:** line joining the camera centers
- **Epipole:** point of intersection of baseline with image plane
- **Epipolar plane:** plane containing baseline and world point
- **Epipolar line:** intersection of epipolar plane with the image plane
- All epipolar lines intersect at the epipole
- An epipolar plane intersects the left and right image planes in epipolar lines



Example: converging cameras

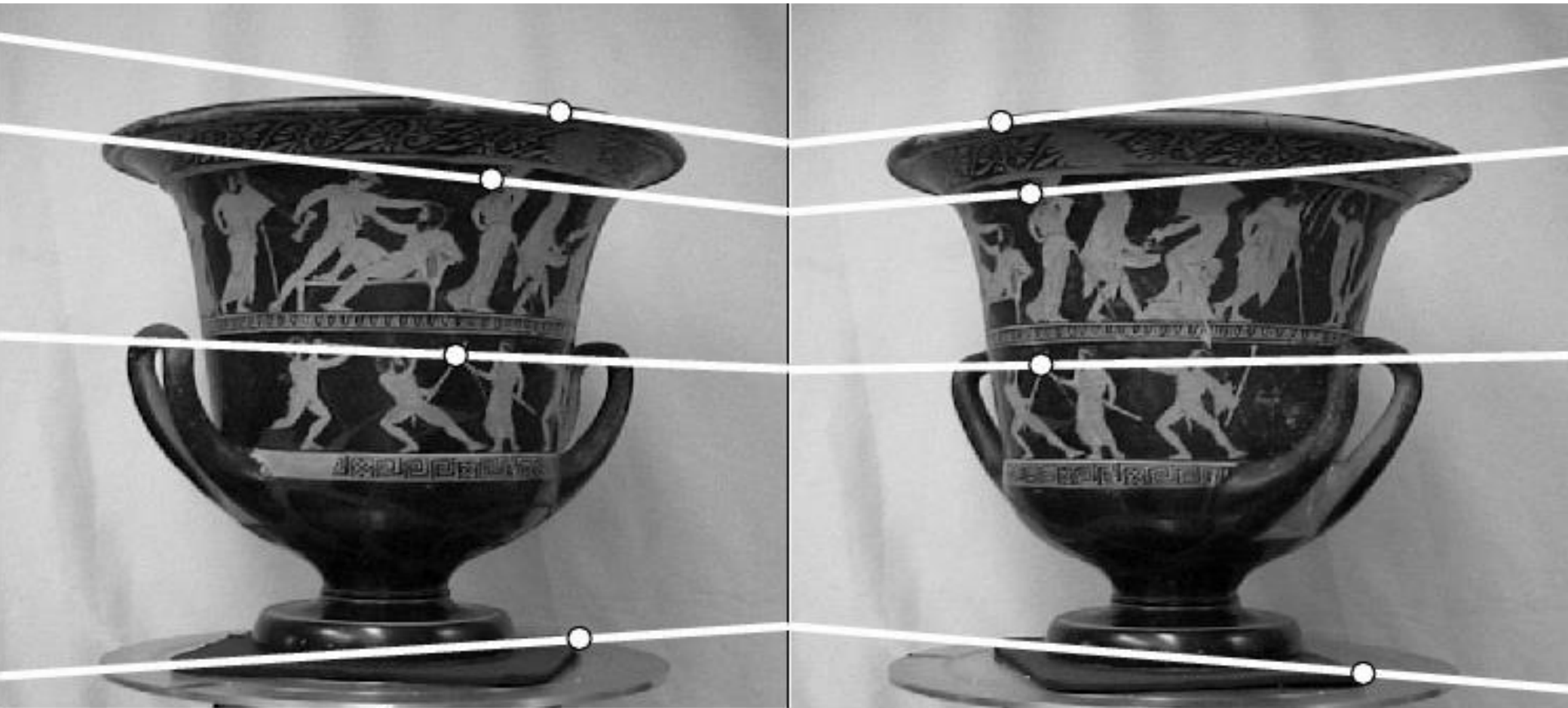
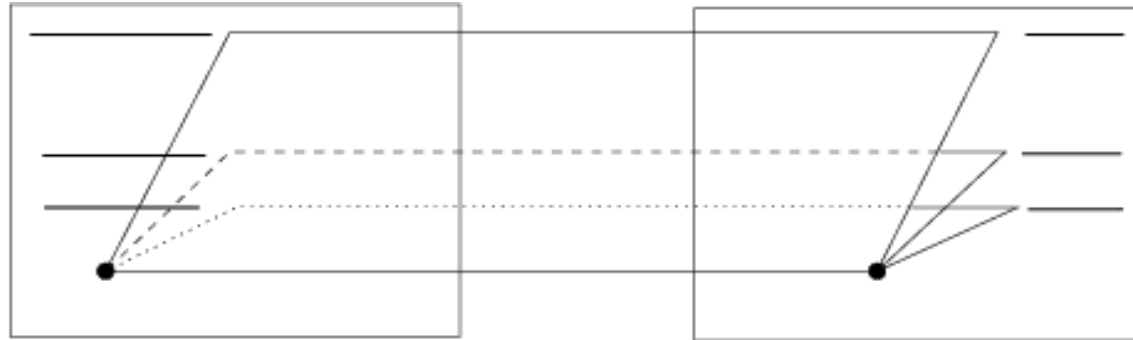


Figure from Hartley & Zisserman

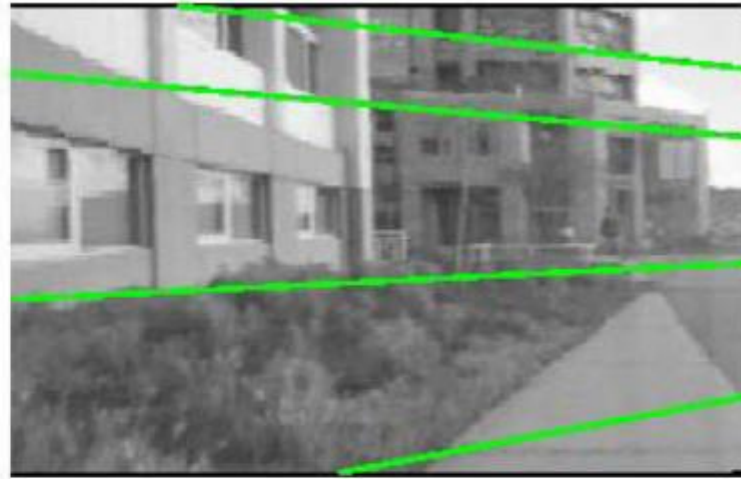
Example: parallel cameras



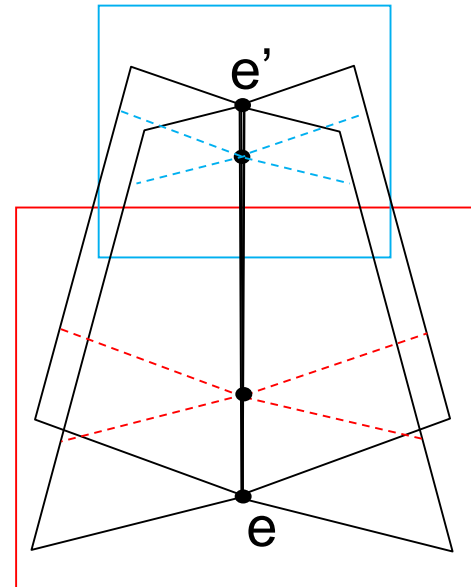
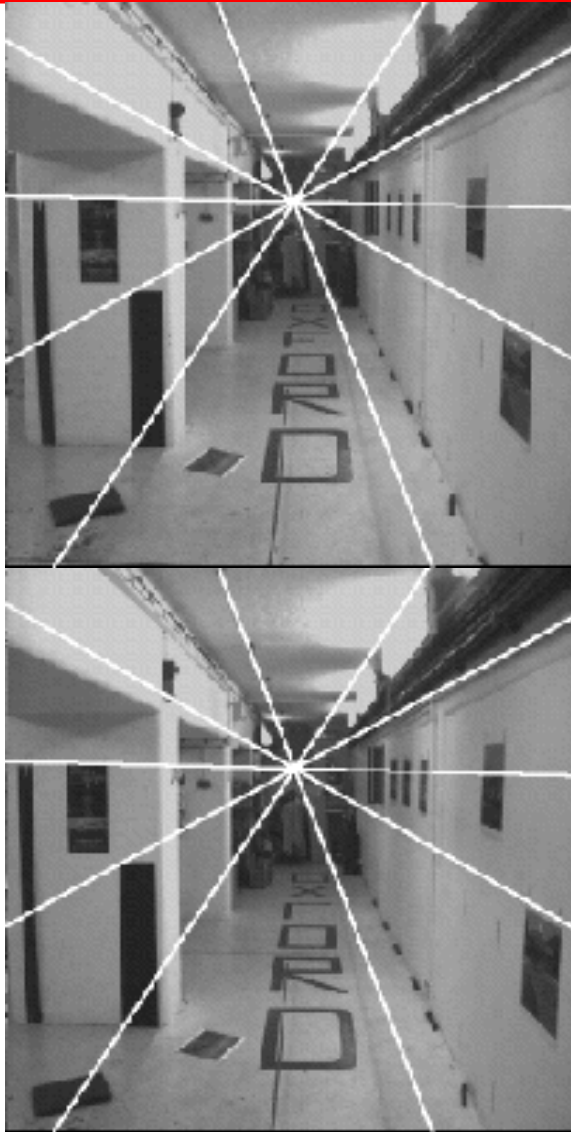
Where are
the
epipoles?



Epipolar constraint example



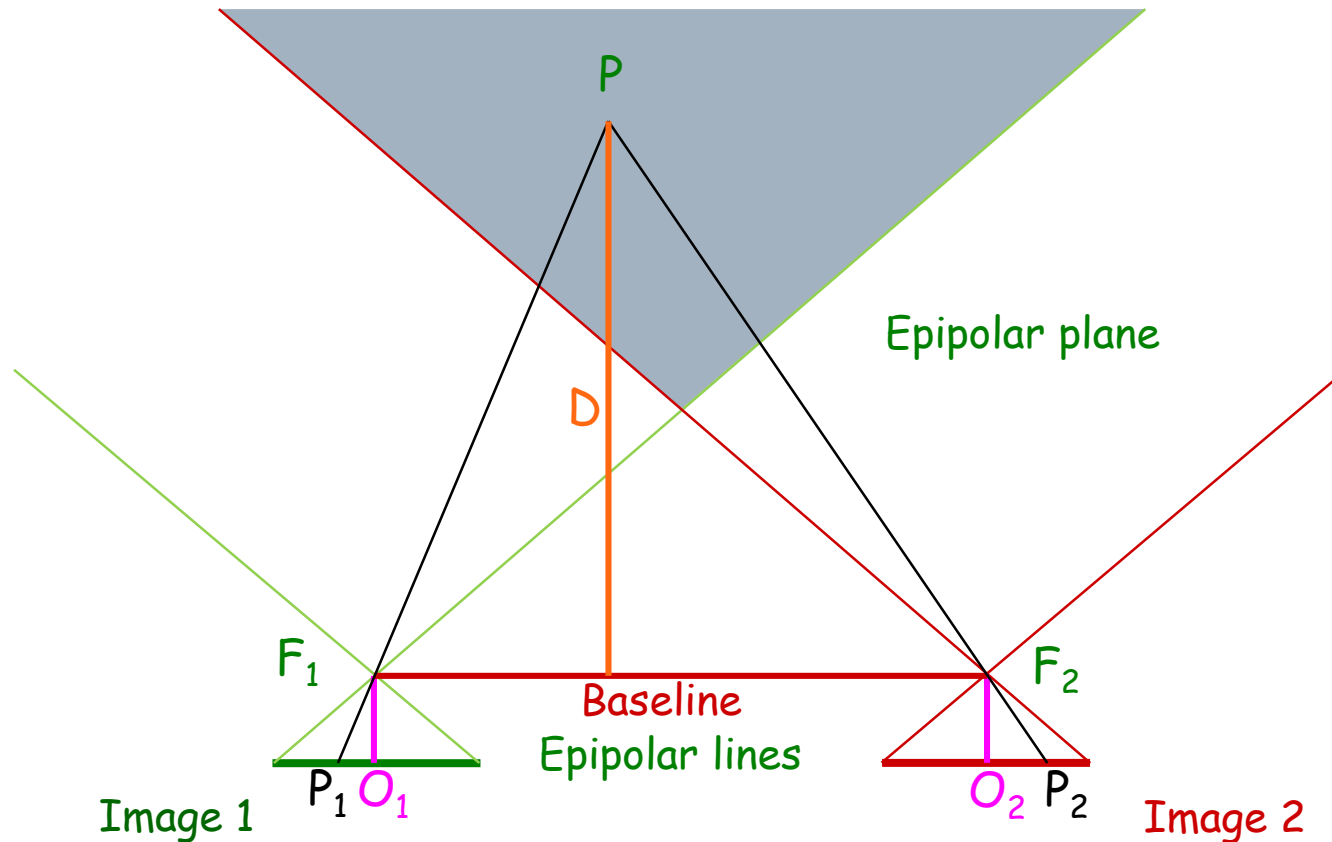
Example: Forward motion



Epipole has same coordinates in both images.
Points move along lines radiating from e :
"Focus of expansion"

Correspondences - homologous points

- Stereo vision geometry: the light gray zone corresponds to the two view-points image overlapping area



Finding the D value

$$\frac{\overleftrightarrow{P_1 O_1 O_2 P_2}}{B} = \frac{f}{D}$$

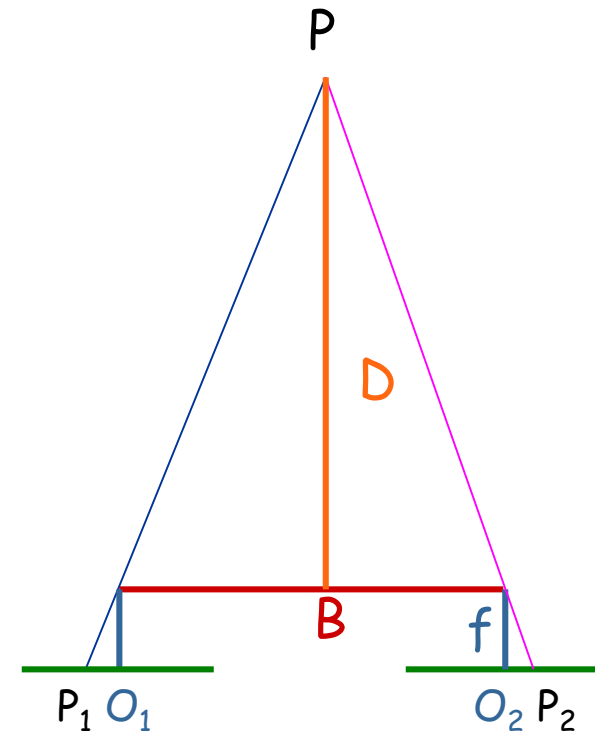
$$D = \frac{f B}{\Delta_1 + \Delta_2}$$

$\Delta_1 + \Delta_2$ displacements on the epipolar lines

- The influence of the distance D on the error of the computed $\Delta = \Delta_1 + \Delta_2$ is evidenced by mere derivation:

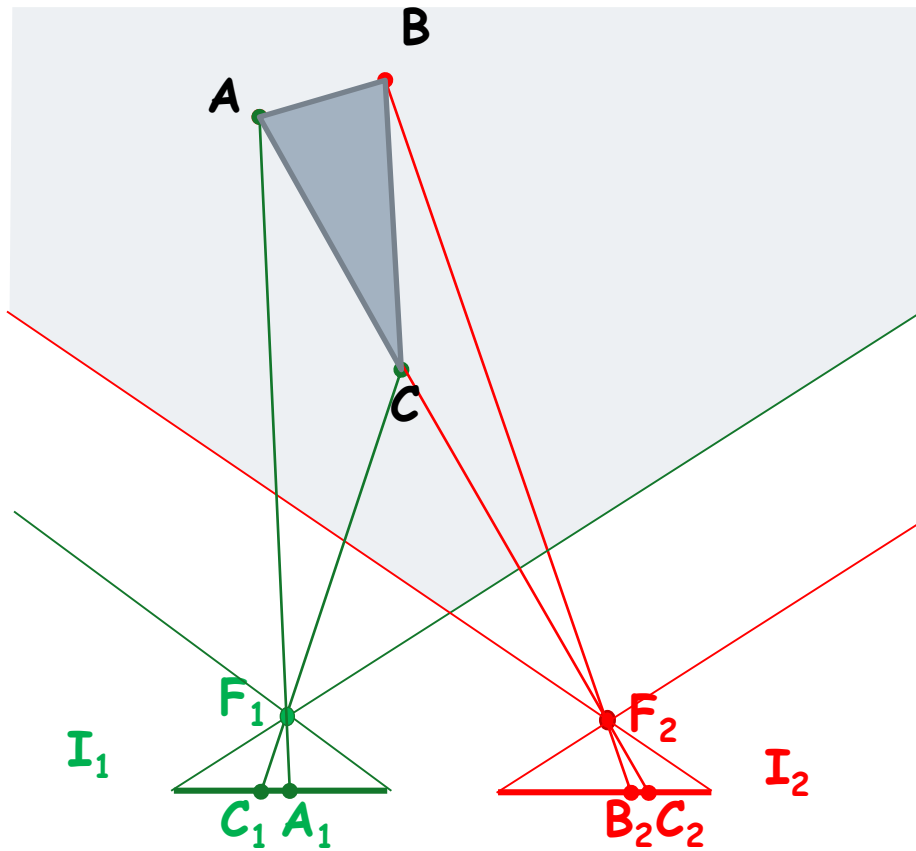
$$\frac{\partial D}{\partial \Delta} = -\frac{D}{\Delta}$$

- Note that the **error increases with the depth** and is **amplified reducing Δ** .

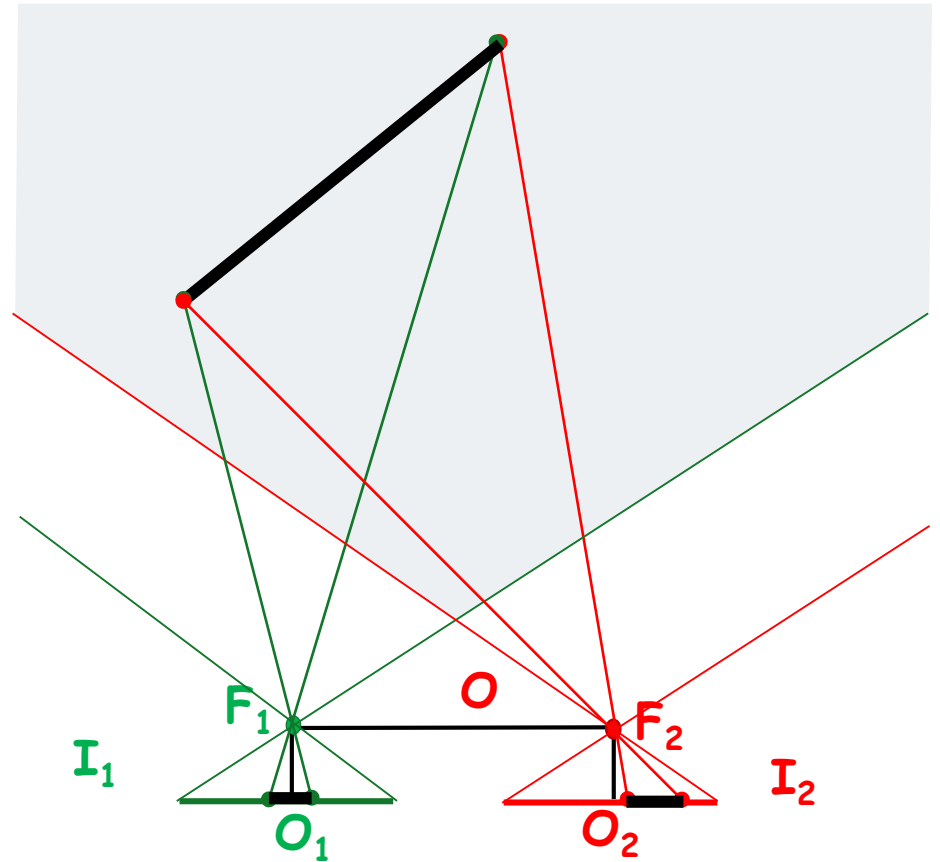


Looking for the tie point

Occlusions : B is occluded in I_1 , A in I_2



Distorted views due to different projections

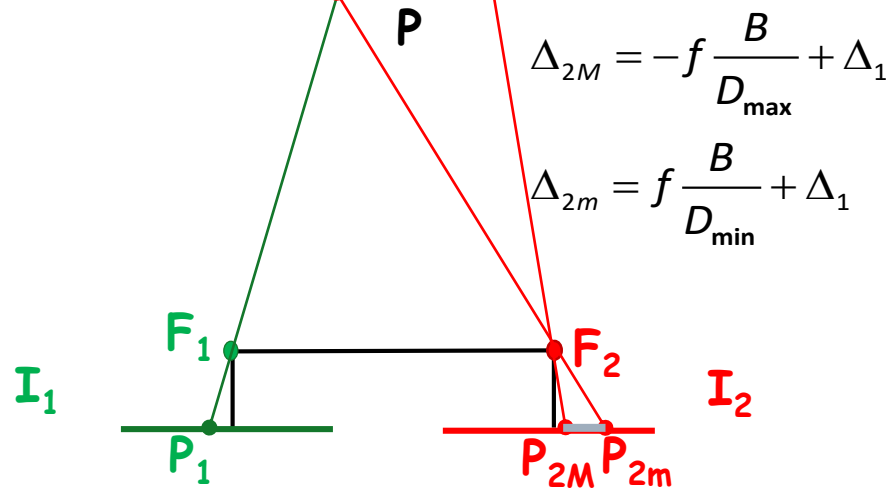


Looking for the tie point

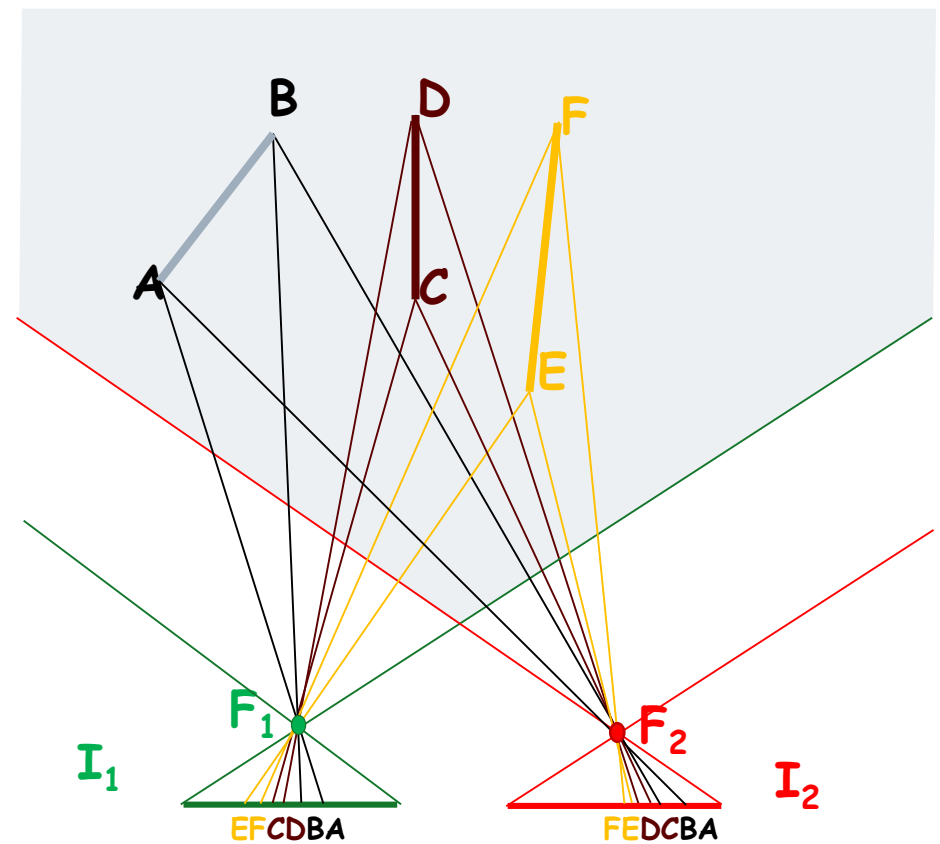
The epipolar segment $P_{2M}P_{2m}$

Maximum distance

Minimum distance

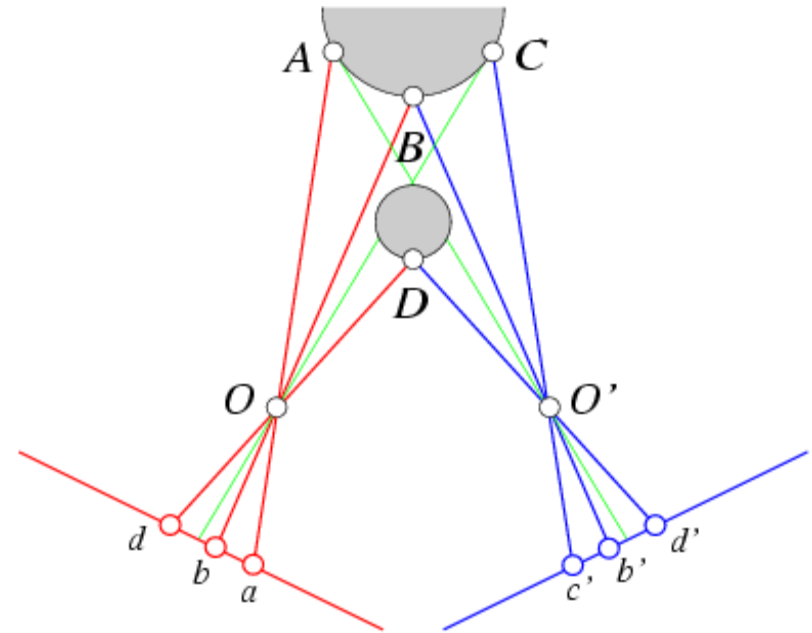
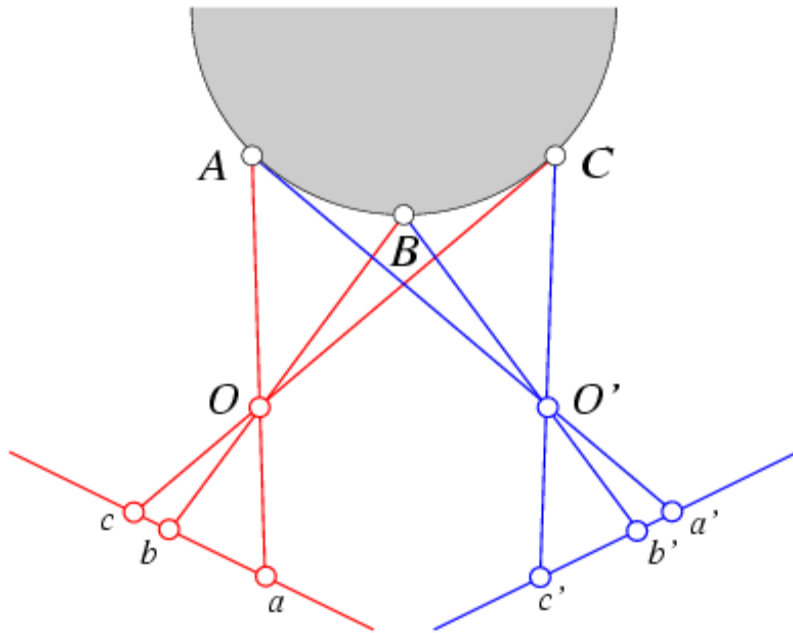


The ordering problem as seen by the letter sequence on each image



Non-local constraints

- Ordering: corresponding points could not be in the same order in both views

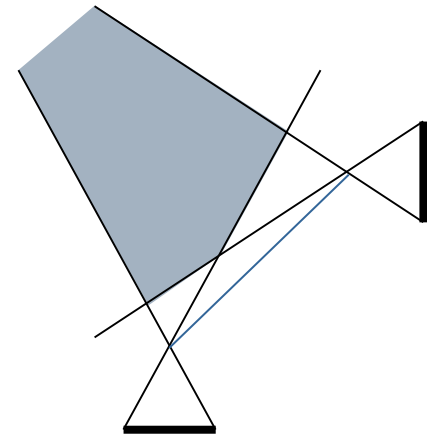
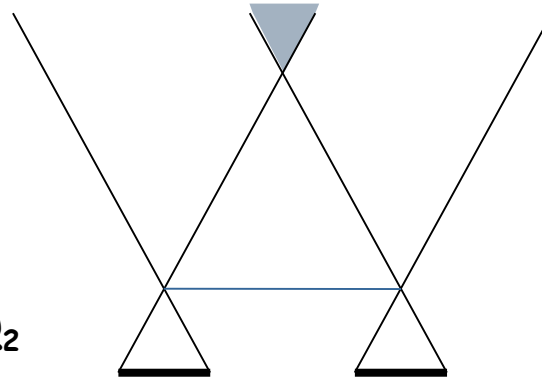
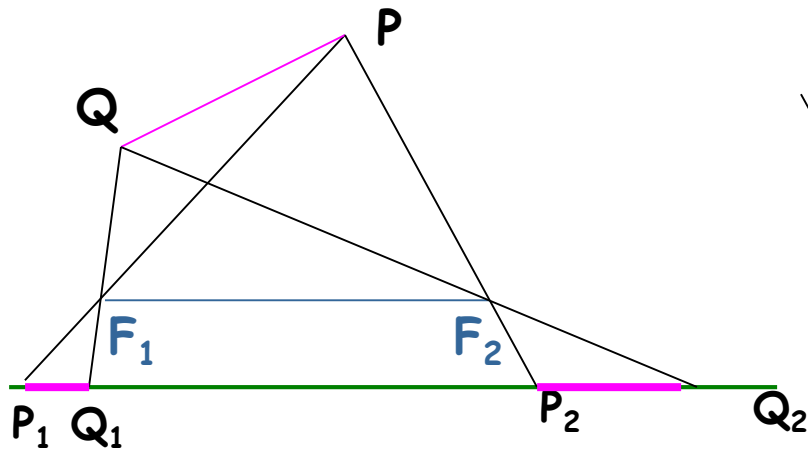


Ordering constraint doesn't hold

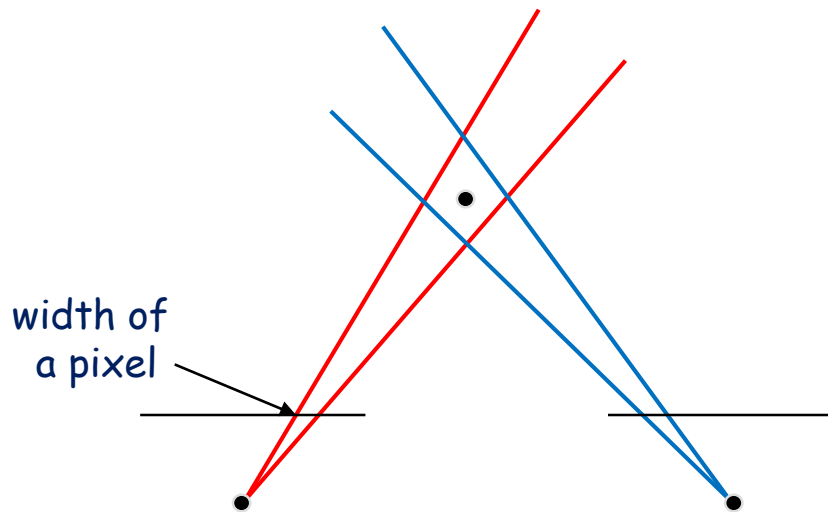
Looking for the tie point

The higher the baseline the higher the deformation and the lower the overlapping

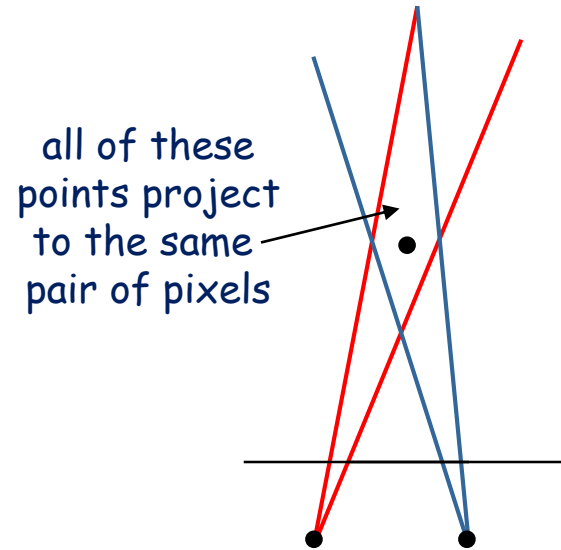
To obtain a useful and extended overlapping area it is often necessary to tilt the camera axis



Choosing the stereo baseline



Large Baseline



Small Baseline

- What's the optimal baseline?
 - Too small: large depth error
 - Too large: difficult search problem

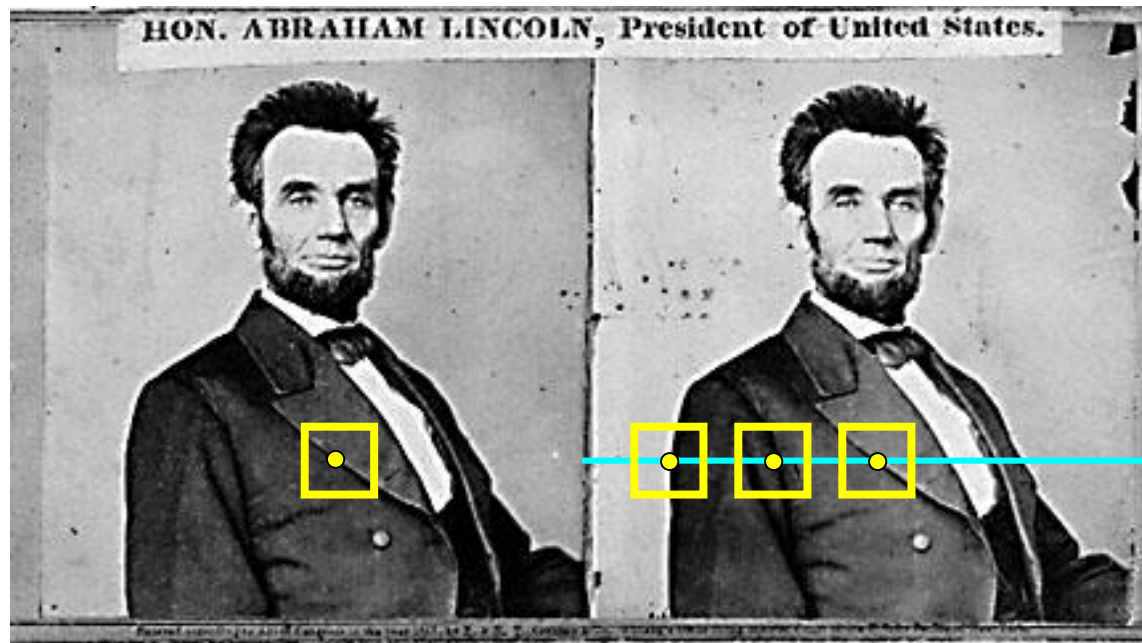
Homologous points

- The simplest ways to determine if a given pixel (p, q) on one image I_1 is a good candidate, is to evaluate the gray level variance in a limited neighborhood of such pixel.
- If its value exceeds a given threshold, then a neighborhood $(2n+1) \times (2m+1)$ is considered and correlated with candidate regions on image I_2 .
- Candidate regions are selected on the epipolar line; in order to compute the correlation between regions of both images the following formula may be used:

$$C(i, j) = \sum_{r=-n}^n \sum_{s=-m}^m [I_2(i+r, j+s) - I_1(p+r, q+s)]^2$$

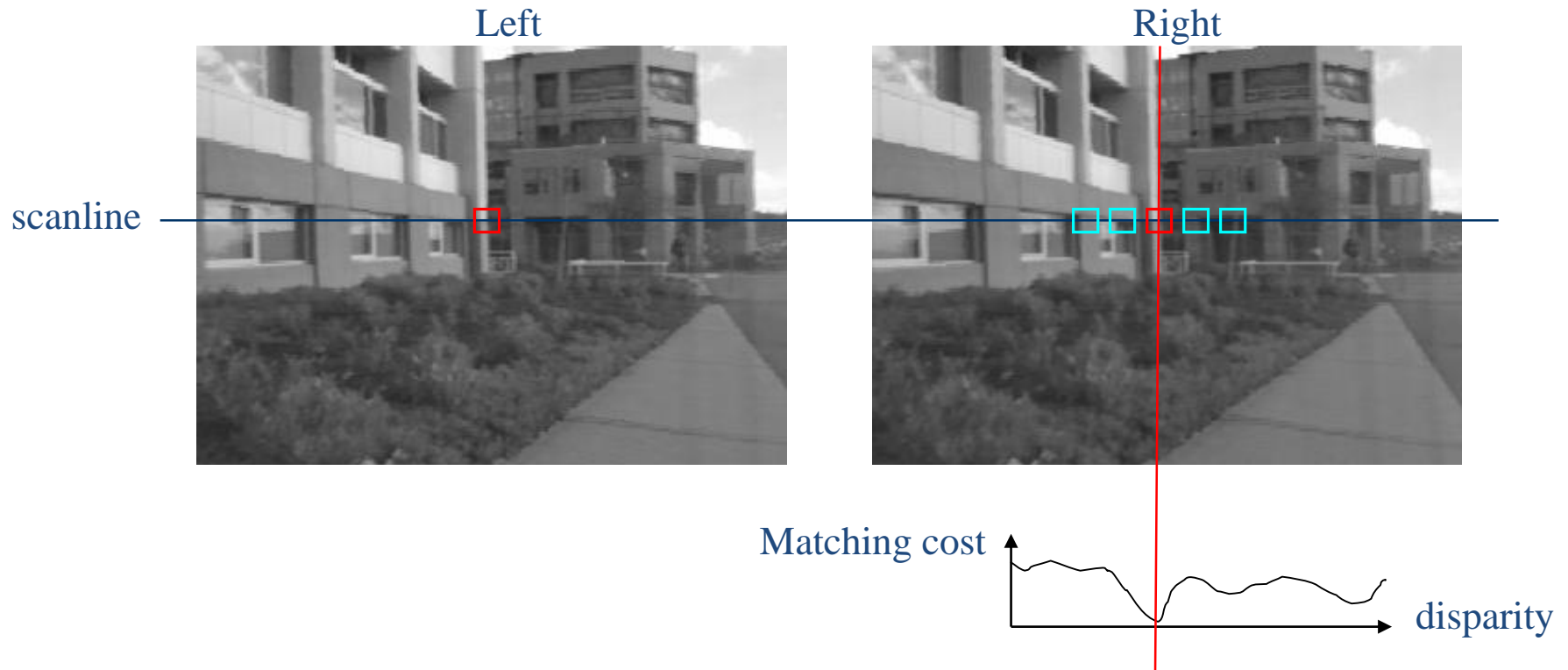
- If cameras are parallel and at the same height, the searching homologous tie points are positioned onto the horizontal epipolar lines with same coordinate. In practical applications only a calibration phase and image registration guarantee such properties.
- A cross check can be applied: if P is obtained from Q , Q must correspond be obtained from P

Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
 - For each pixel x in the first image
 - Find corresponding epipolar scanline in the right image
 - Examine all pixels on the scanline and pick the best match x'
 - Compute disparity $x-x'$ and set $\text{depth}(x) = fB/(x-x')$
-

Correspondence search



- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
 - Matching cost: SSD or normalized correlation
-

Exemple



Matching windows

Similarity Measure

Sum of Absolute Differences (SAD)

Sum of Squared Differences (SSD)

Zero-mean SAD

Locally scaled SAD

Normalized Cross Correlation (NCC)

Formula

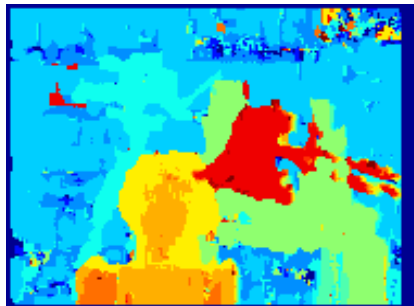
$$\sum_{(i,j) \in W} |I_1(i,j) - I_2(x+i, y+j)|$$

$$\sum_{(i,j) \in W} (I_1(i,j) - I_2(x+i, y+j))^2$$

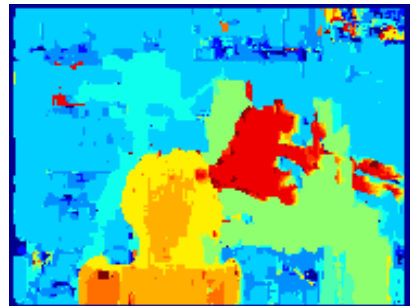
$$\sum_{(i,j) \in W} |I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i, y+j) + \bar{I}_2(x+i, y+j)|$$

$$\sum_{(i,j) \in W} |I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i, y+j)} I_2(x+i, y+j)|$$

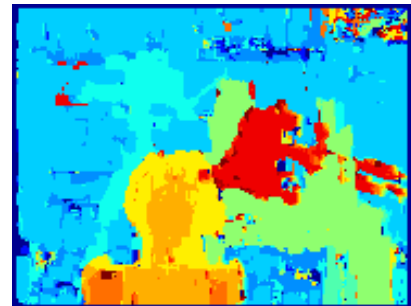
$$\frac{\sum_{(i,j) \in W} I_1(i,j) \cdot I_2(x+i, y+j)}{\sqrt{\sum_{(i,j) \in W} I_1^2(i,j) \cdot \sum_{(i,j) \in W} I_2^2(x+i, y+j)}}$$



SAD



SSD

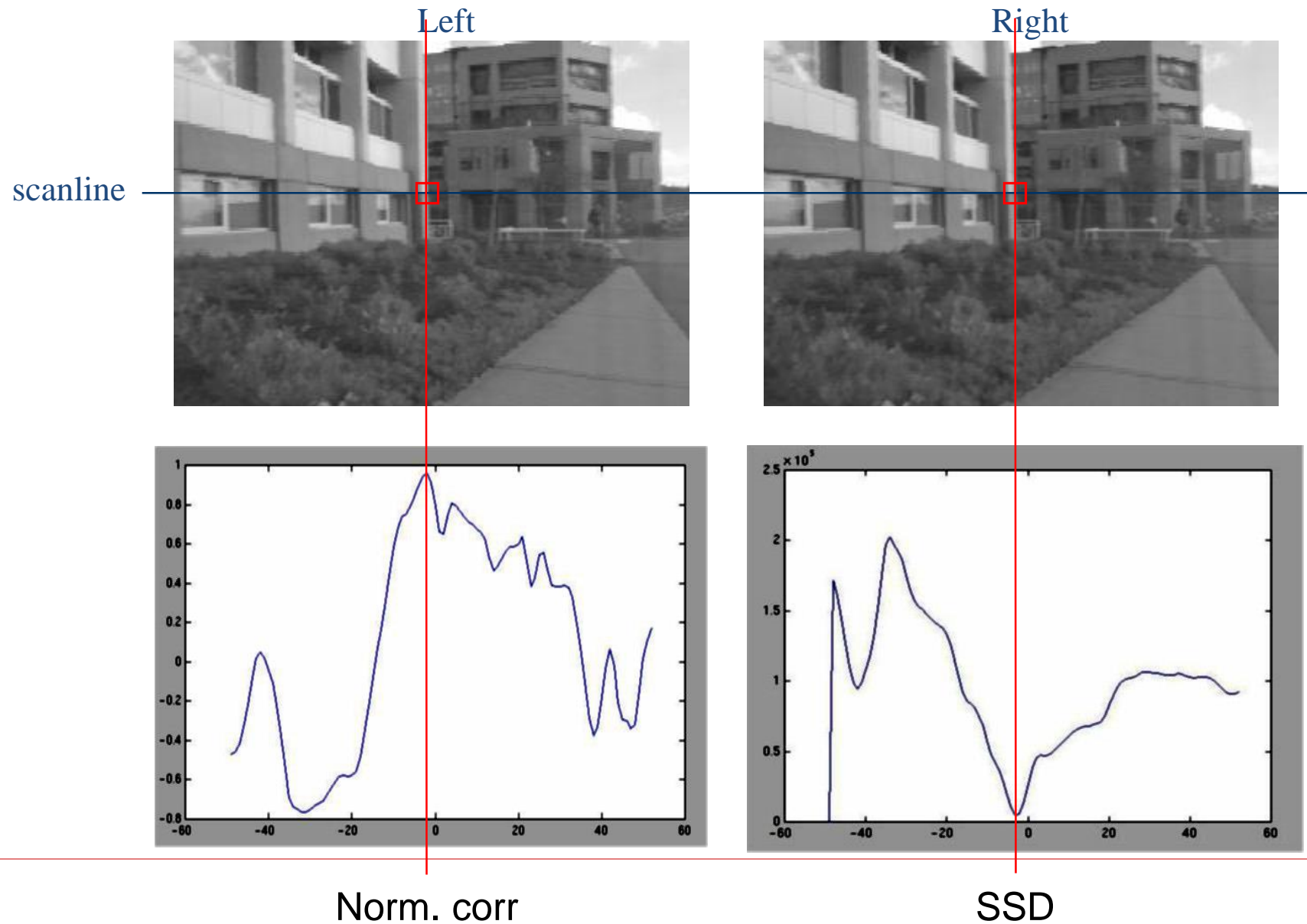


NCC



Ground truth

Correspondence search



Implementation aspects

The search can be done in four steps:

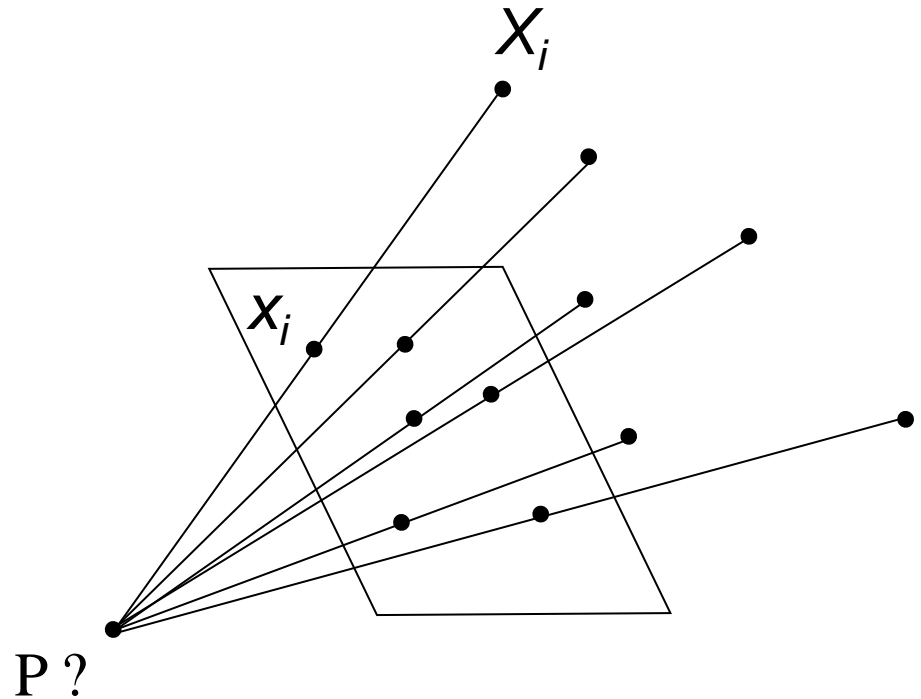
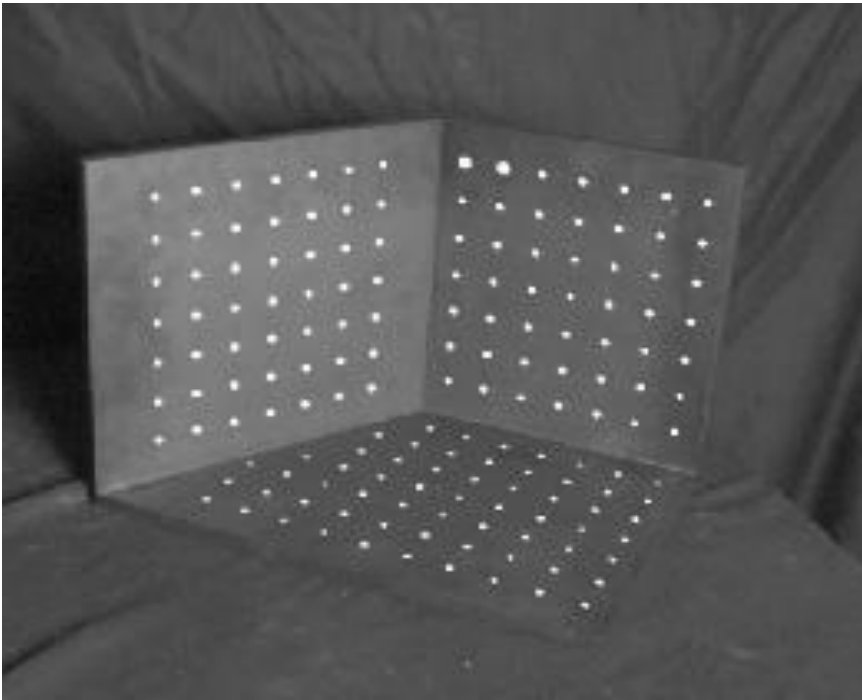
- **Selection of interesting points** (through a threshold S_1 applied to the variance in the neighborhood or to the result of an *edge detector*)
- For each point selected, **finding if exists the tie point** (with a cross-check and a threshold S_2 of cross-similarity)
- **Evaluation of the distance** on the basis of the extracted homologous points

Experimentation of the best solution, considering that:

- augmenting S_1 the number of tie points is reduced but the reliability increases
- augmenting S_2 increases the number of homologous couples but it is reduced the reliability

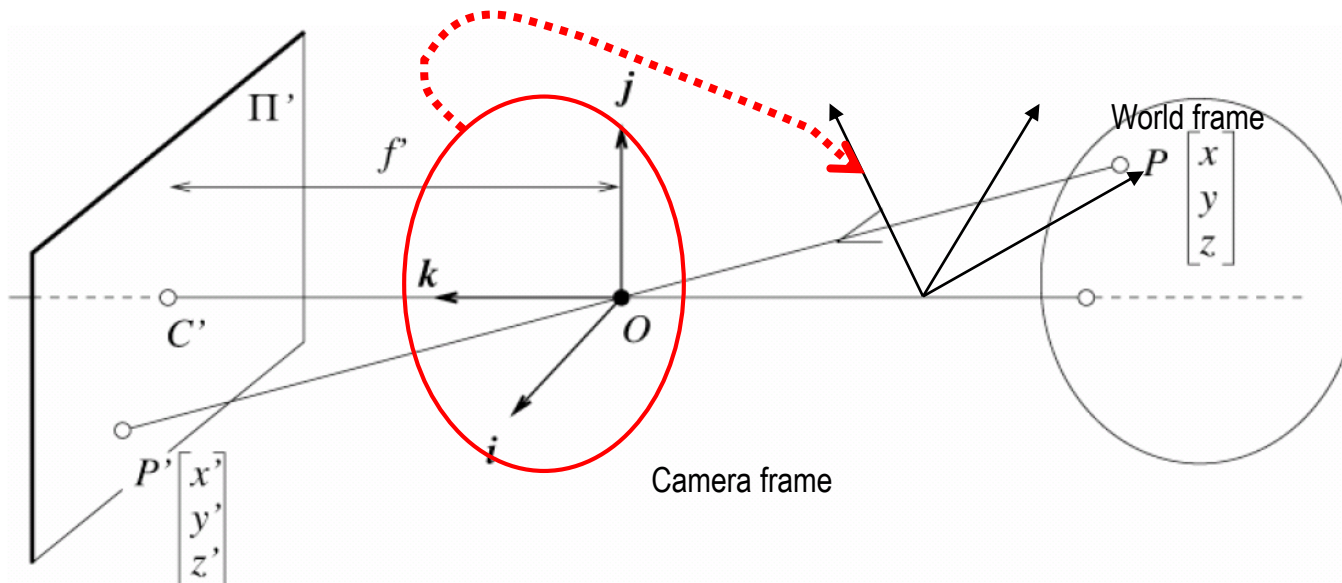
Camera calibration

- Given n points with known 3D coordinates X_i and known image projections x_i , estimate the camera parameters

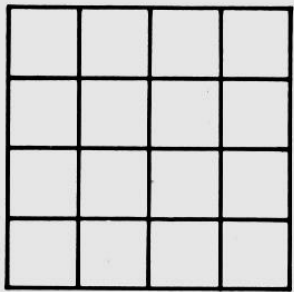


Camera parameters

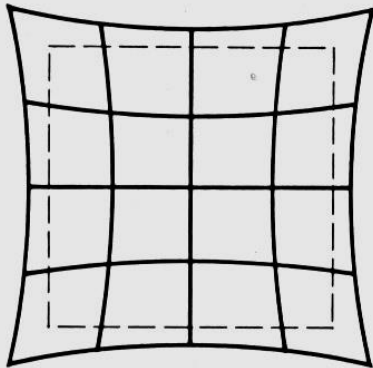
- Intrinsic parameters
 - Focal length
 - Pixel size
 - Radial distortion
- Extrinsic parameters
 - Rotation and translation relative to world coordinate



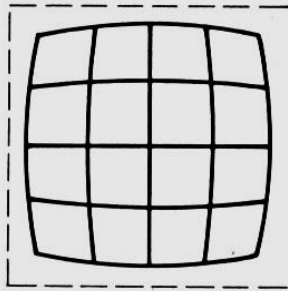
Beyond Pinholes: Radial Distortion



No distortion



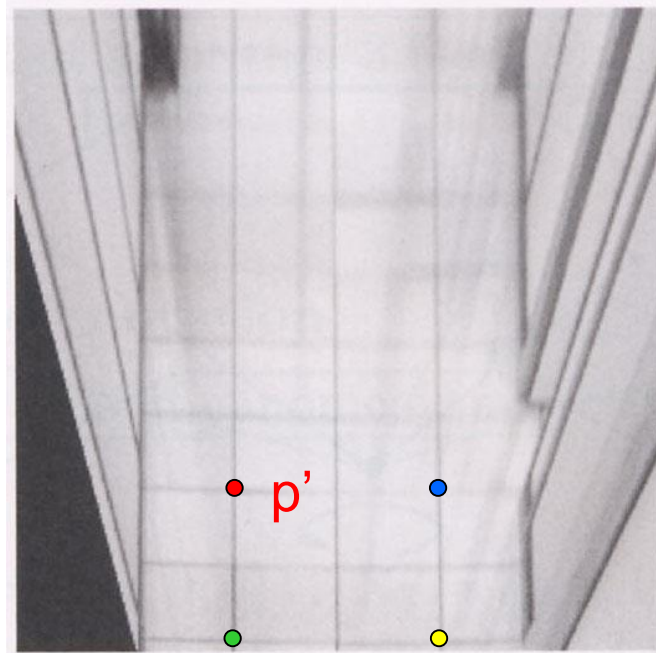
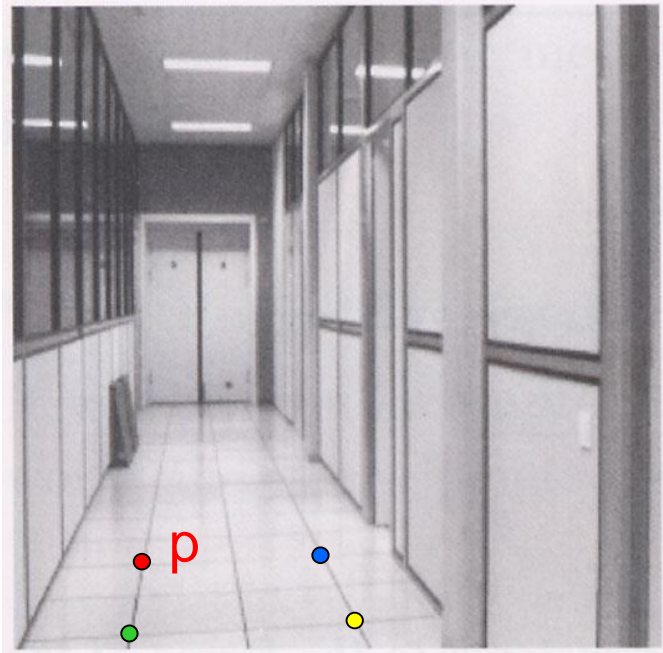
Pin cushion



Barrel



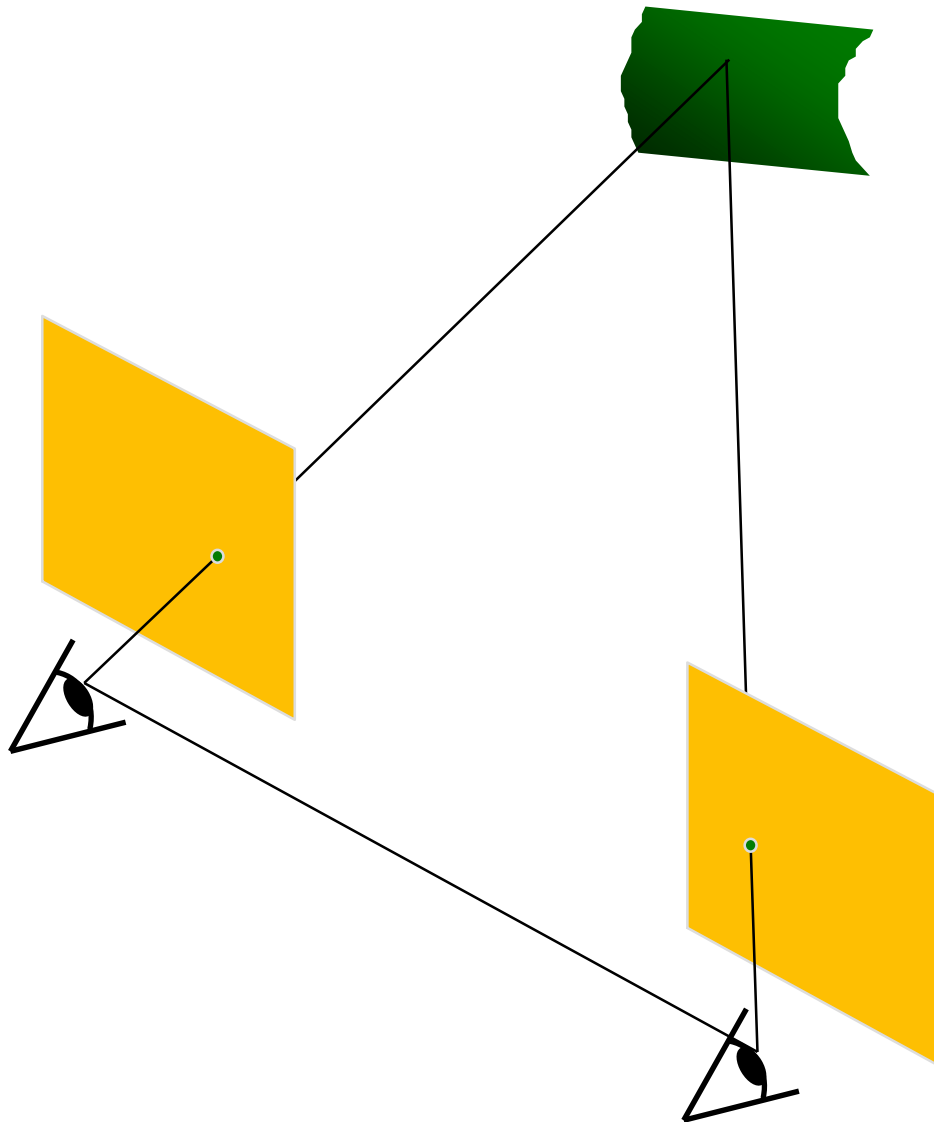
Image rectification



To unwarp (rectify) an image

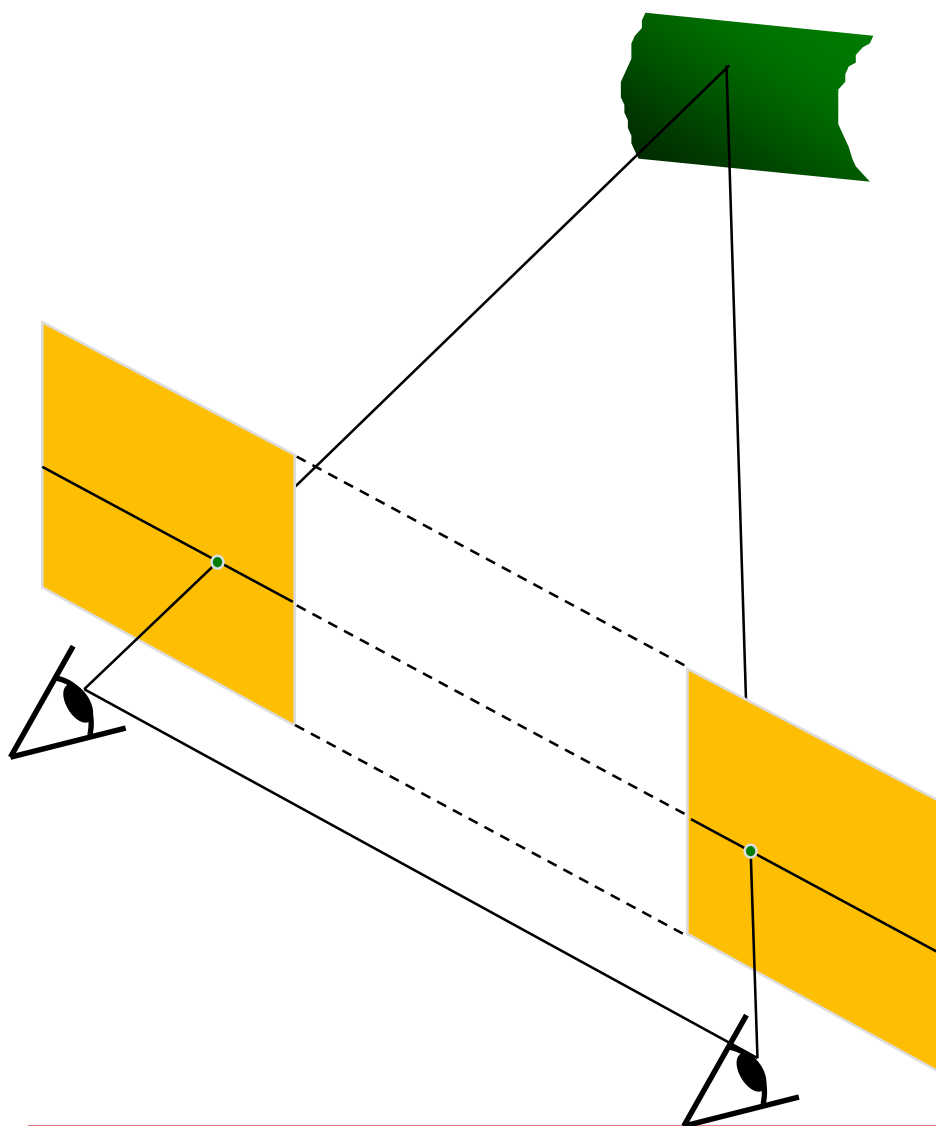
- solve for homography H given p and p'
- solve equations of the form: $p' = Hp$
 - linear in unknowns: coefficients of H

Simplest Case: Parallel images



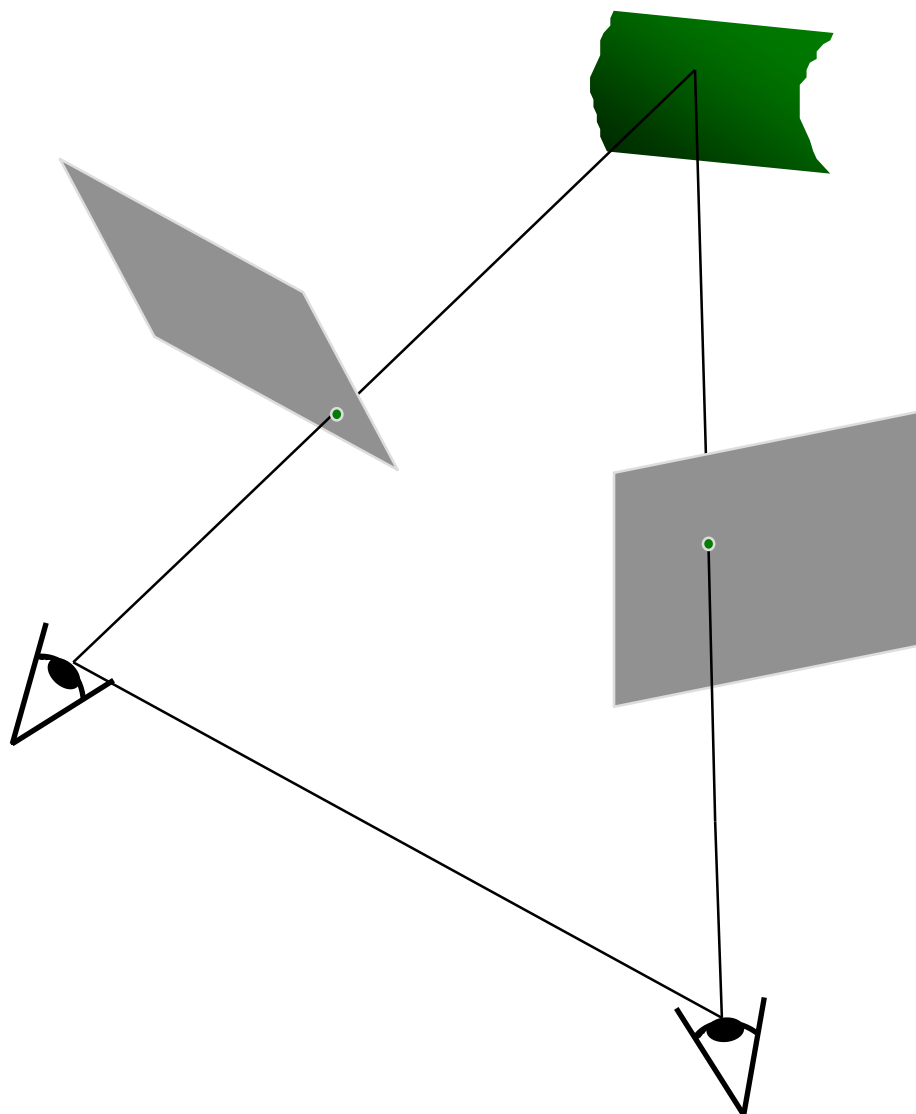
- Image planes of cameras are parallel to each other and to the baseline
 - Camera centers are at same height
 - Focal lengths are the same
-

Simplest Case: Parallel images



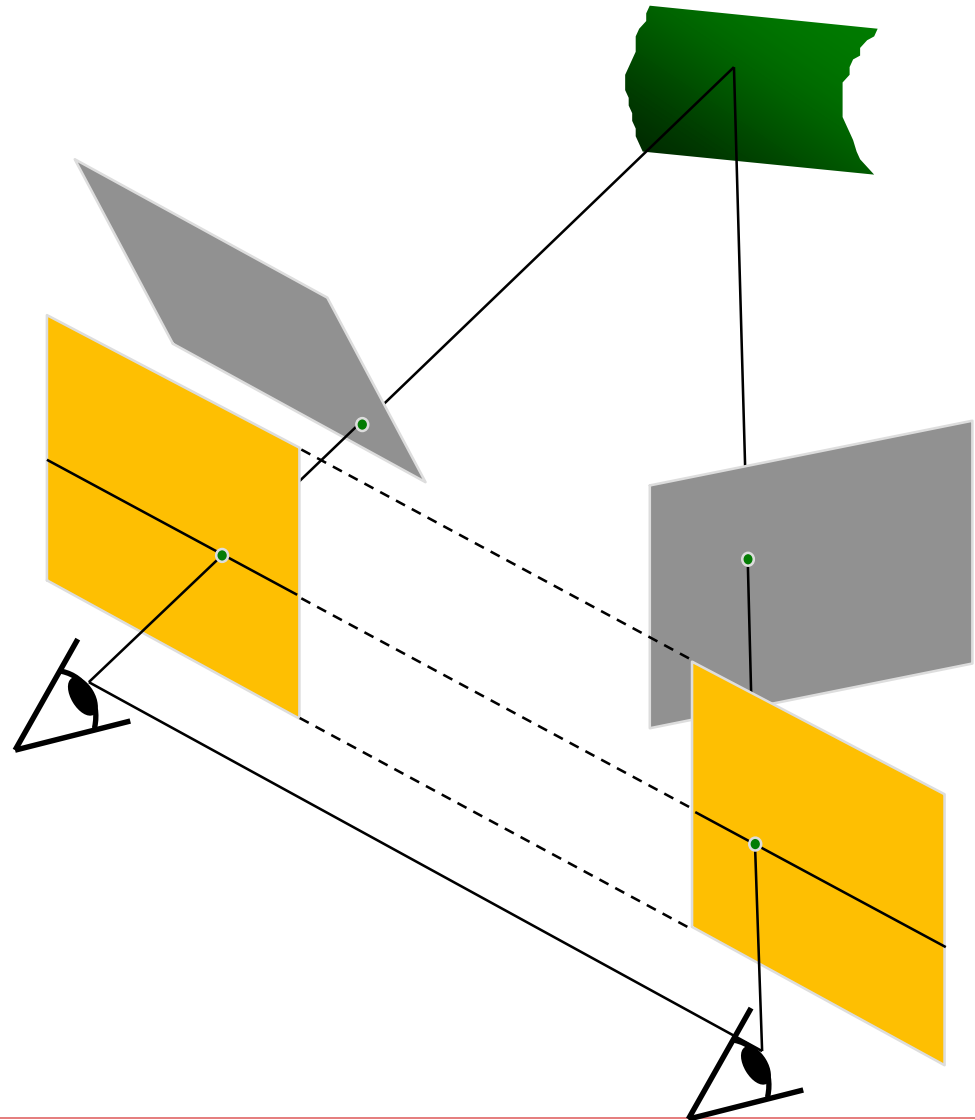
- Image planes of cameras are parallel to each other and to the baseline
 - Camera centers are at same height
 - Focal lengths are the same
 - Then, epipolar lines fall along the horizontal scan lines of the images
-

Stereo image rectification

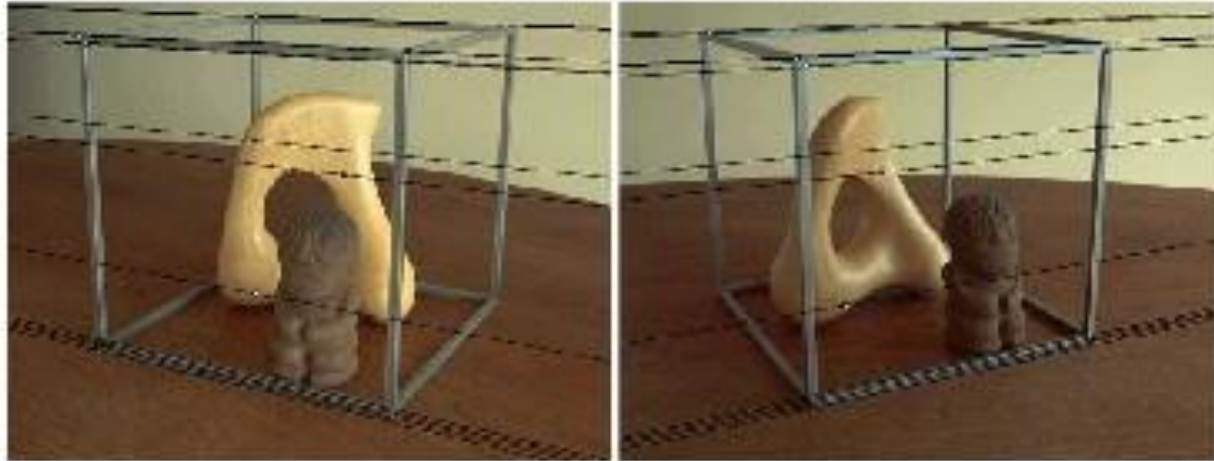


Stereo image rectification

- reproject image planes onto a common plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- two homographies (3x3 transform), one for each input image reprojection
- C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](#). IEEE Conf. Computer Vision and Pattern Recognition, 1999.



Rectification example



Example

Unrectified



Rectified



Binocular stereo

- Given a calibrated binocular stereo pair, fuse it to produce a depth image

image 1



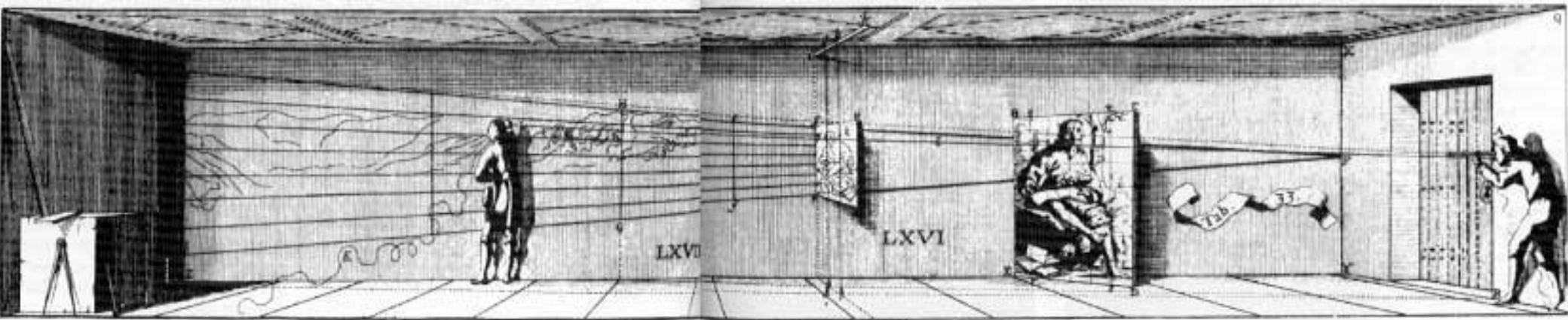
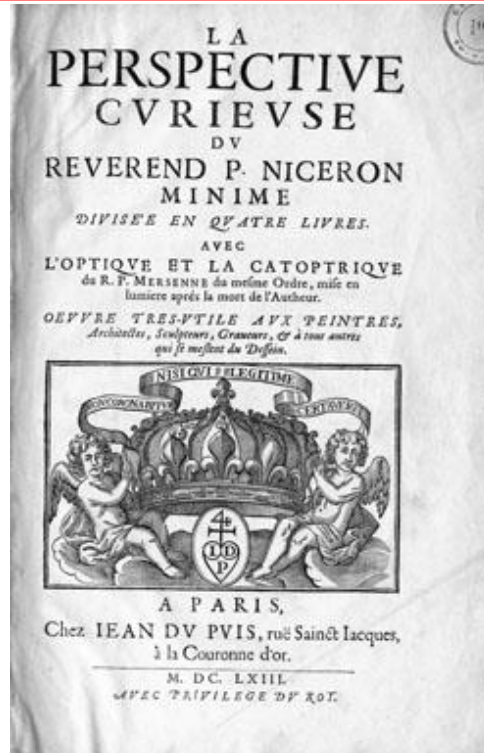
image 2



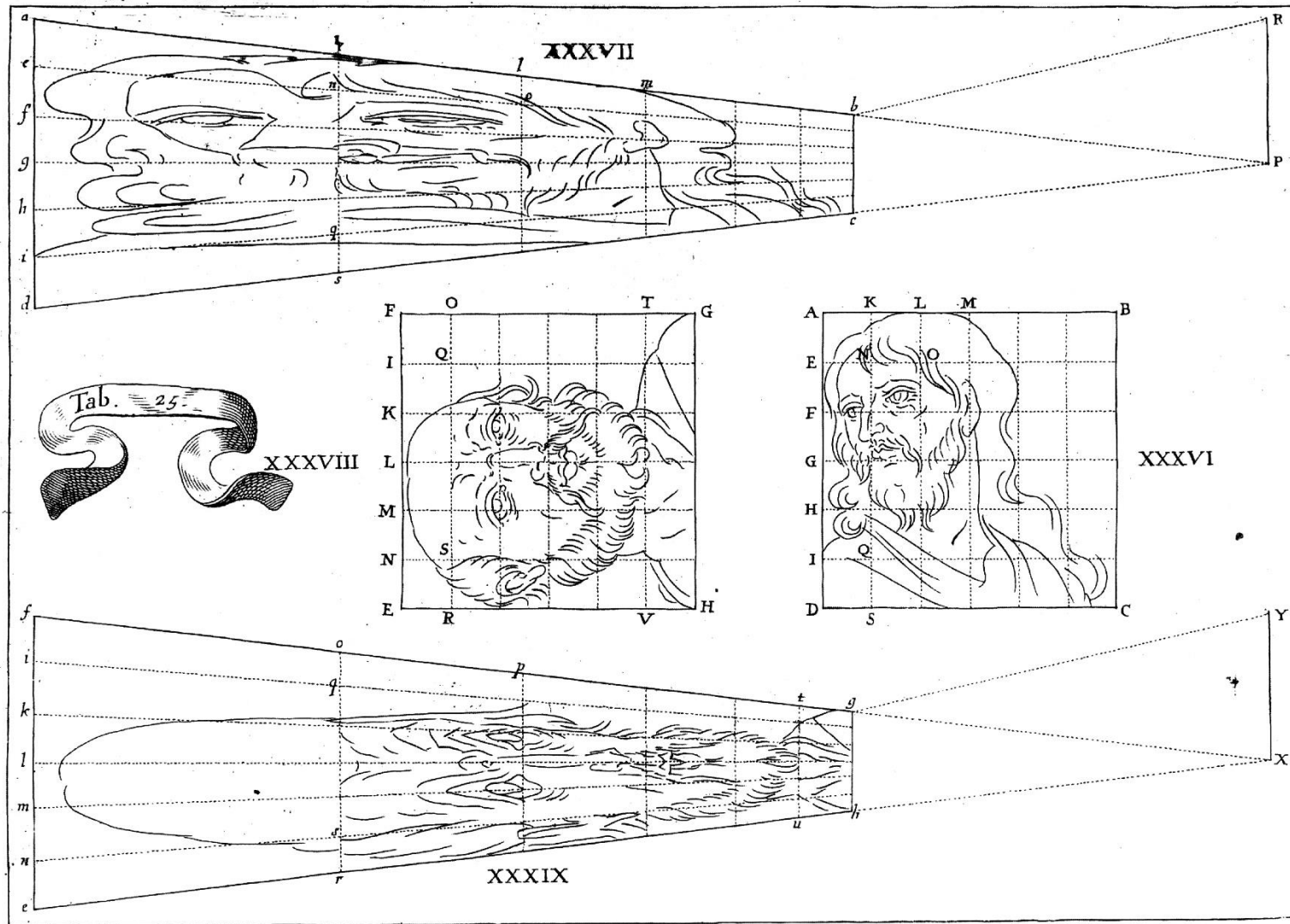
Dense depth map



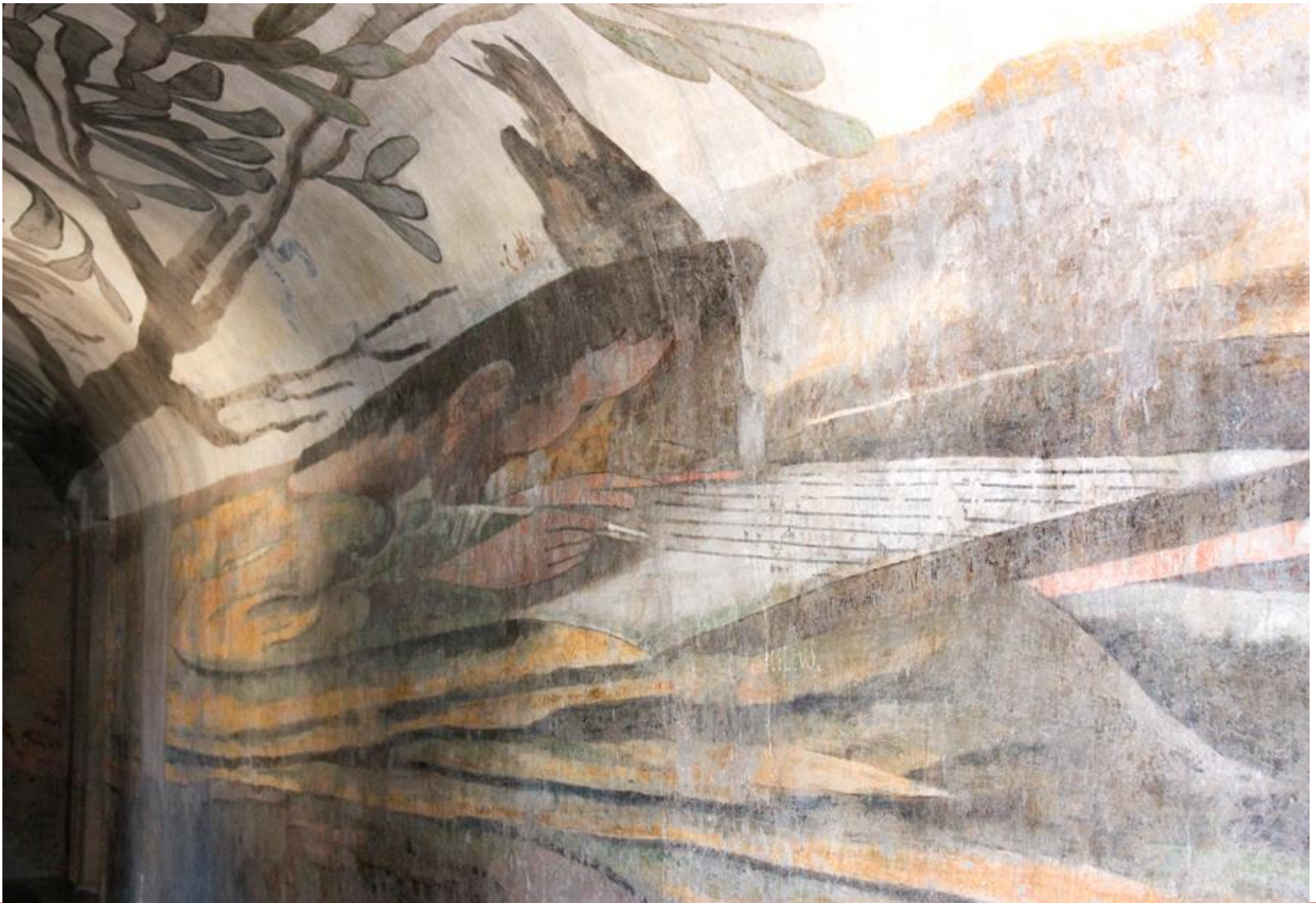
How to make an anamorphic projection



Jean-François Nicéron



Jean-François Nicéron, *San Giovanni evangelista nell'isola di Patmo*
Convento della SS.Trinità dei Monti, Roma 1639



Jean-François Nicéron, *San Giovanni evangelista nell'isola di Patmo*



Un'iscrizione in greco di grande impatto, presente nell'area centrale della composizione: "L'APOCALISSE DELL'OTTICA È TESTIMONE

OCULARE DELL'APOCALISSE". Una civetta, simbolo della sapienza, scruta in disparte il grande scenario.



Jean-François Nicéron



Emmanuel Maignan, *San Francesco di Paola in preghiera*, 1642
Convento della SS. Trinità dei Monti, Roma



<https://www.youtube.com/watch?v=4aGWQOzH0JY>

Multi-view Stereo

Johann Zahn, “the radiating eye”

from *Oculus Artificialis Teledioptricus Sive Telescopium* (1702)

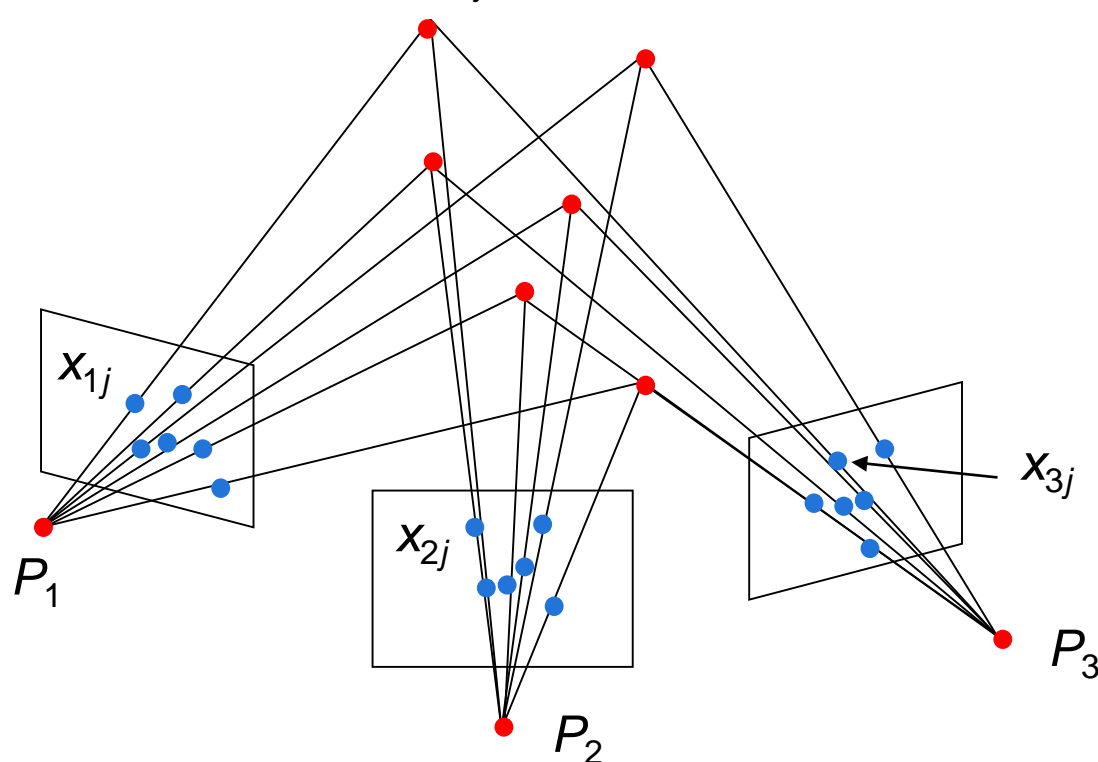


Projective structure from motion

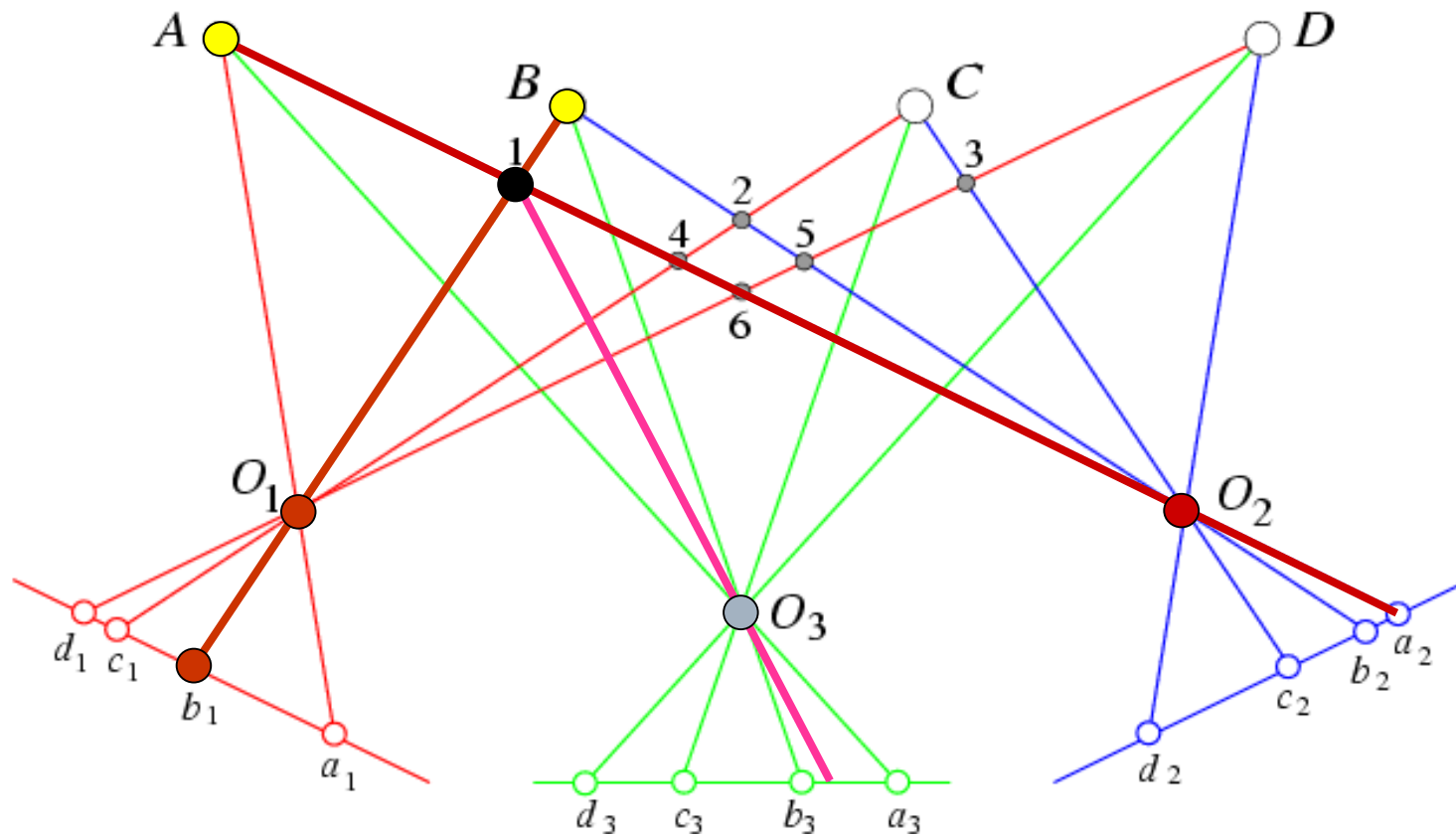
- Given: m images of n fixed 3D points

$$\mathbf{x}_{ij} = \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- Problem: estimate m projection matrices \mathbf{P}_i and n 3D points \mathbf{X}_j from the mn corresponding points \mathbf{x}_{ij}



Beyond two-view stereo

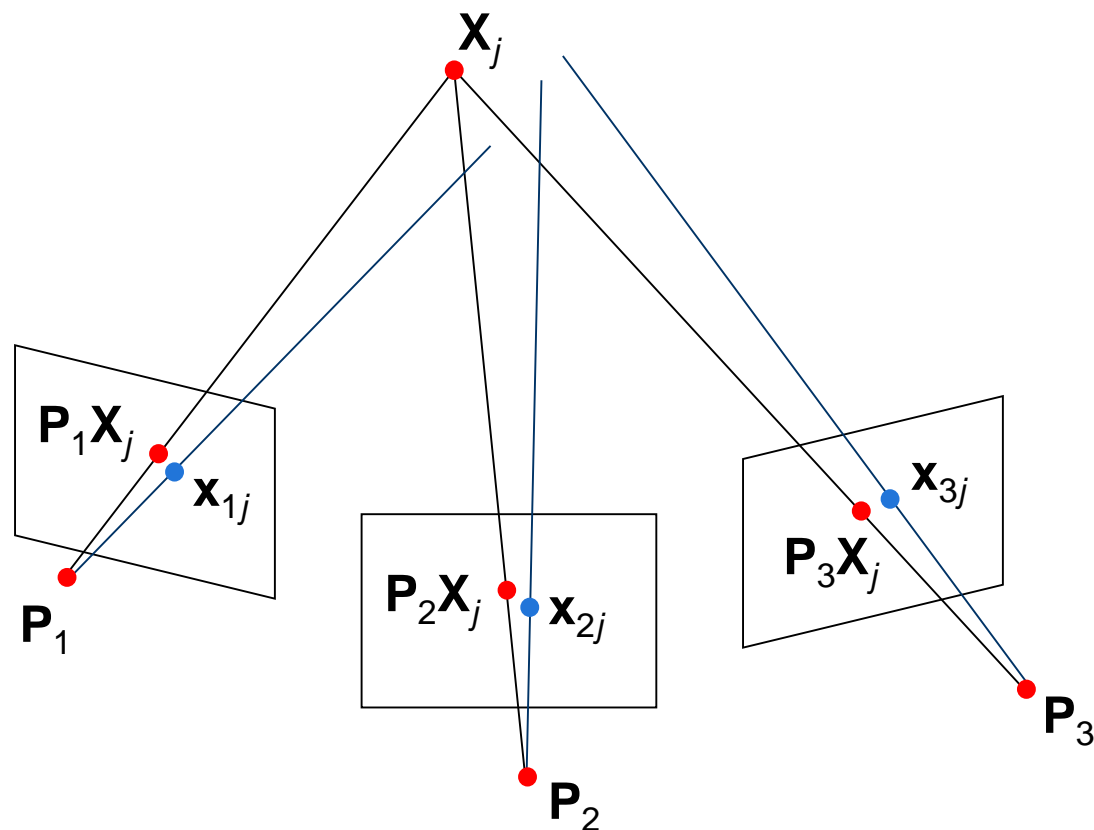


The third view can be used for verification

Bundle adjustment

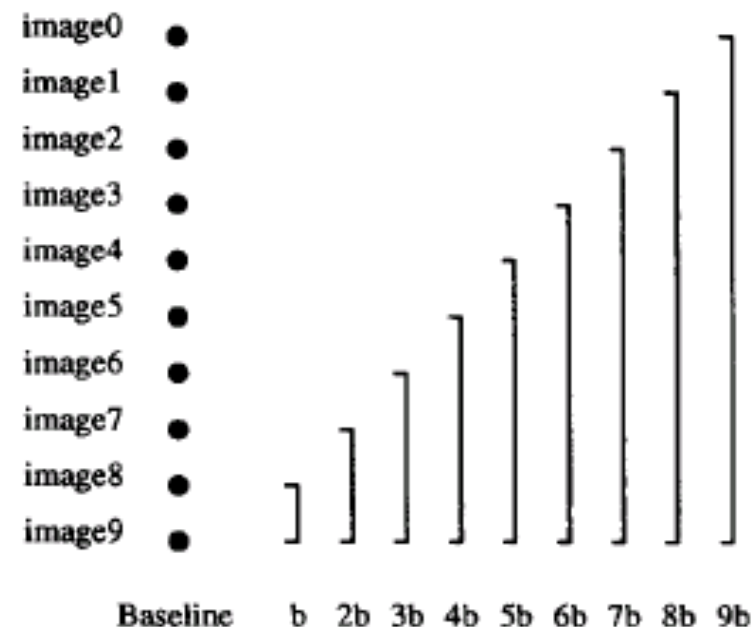
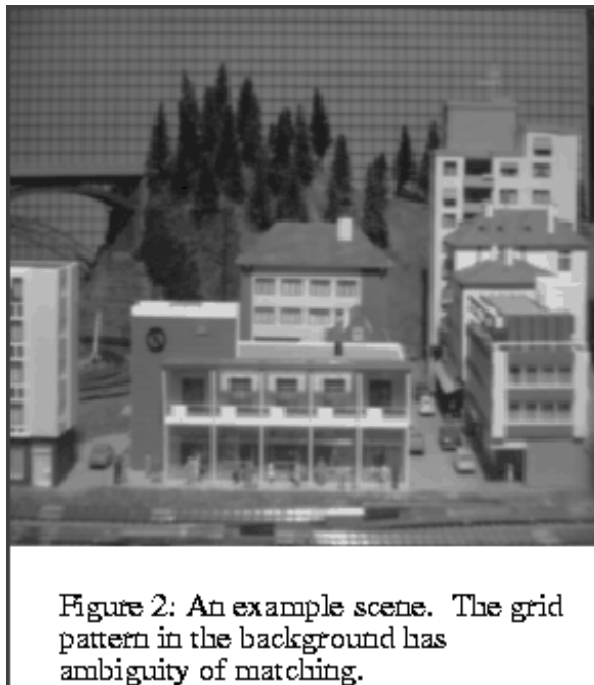
- Non-linear method for refining structure and motion
- Minimizing reprojection error

$$E(\mathbf{P}, \mathbf{X}) = \sum_{i=1}^m \sum_{j=1}^n D(\mathbf{x}_{ij}, \mathbf{P}_i \mathbf{X}_j)^2$$



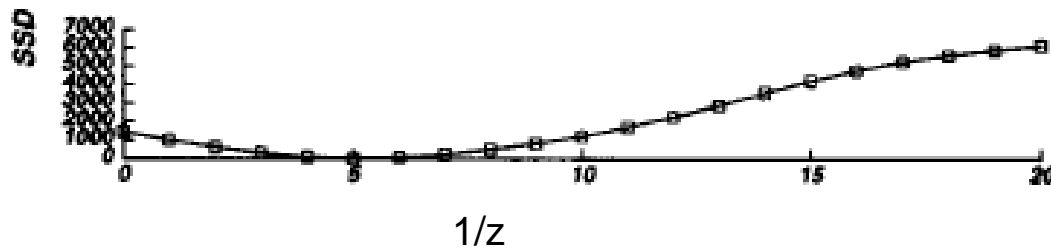
Multiple-baseline stereo

- Pick a reference image, and slide the corresponding window along the corresponding epipolar lines of all other images, using **inverse depth** relative to the first image as the search parameter

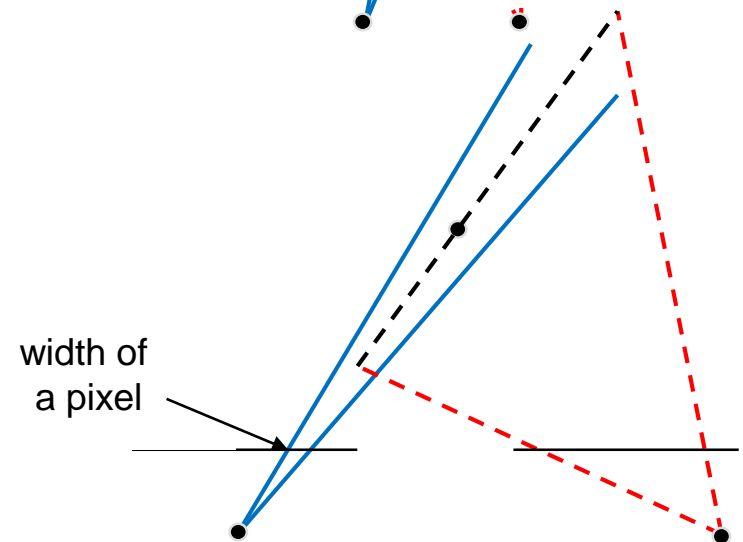
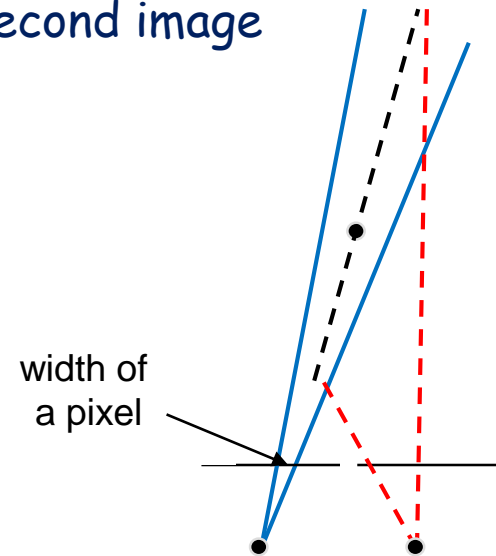
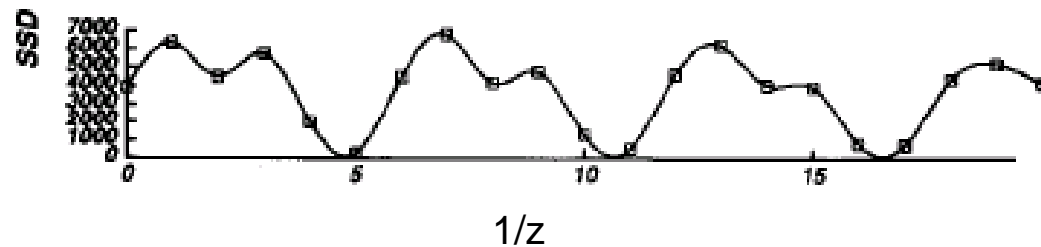


Multiple-baseline stereo

- For larger baselines, must search larger area in second image



pixel matching score



Multiple-baseline stereo

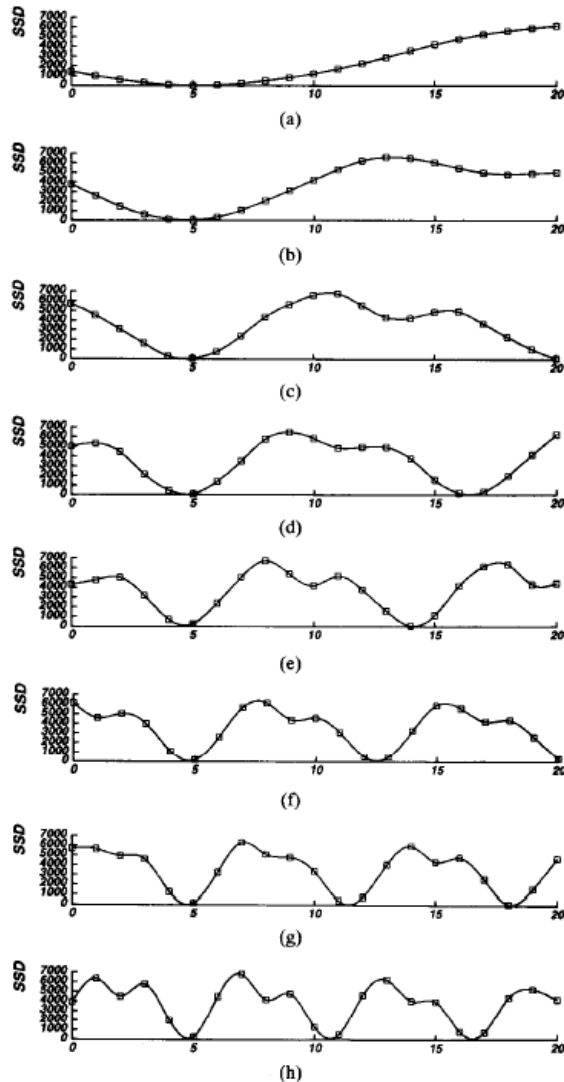


Fig. 5. SSD values versus inverse distance: (a) $B = b$; (b) $B = 2b$; (c) $B = 3b$; (d) $B = 4b$; (e) $B = 5b$; (f) $B = 6b$; (g) $B = 7b$; (h) $B = 8b$. The horizontal axis is normalized such that $8bF = 1$.

Use the sum of SSD
scores to rank
matches

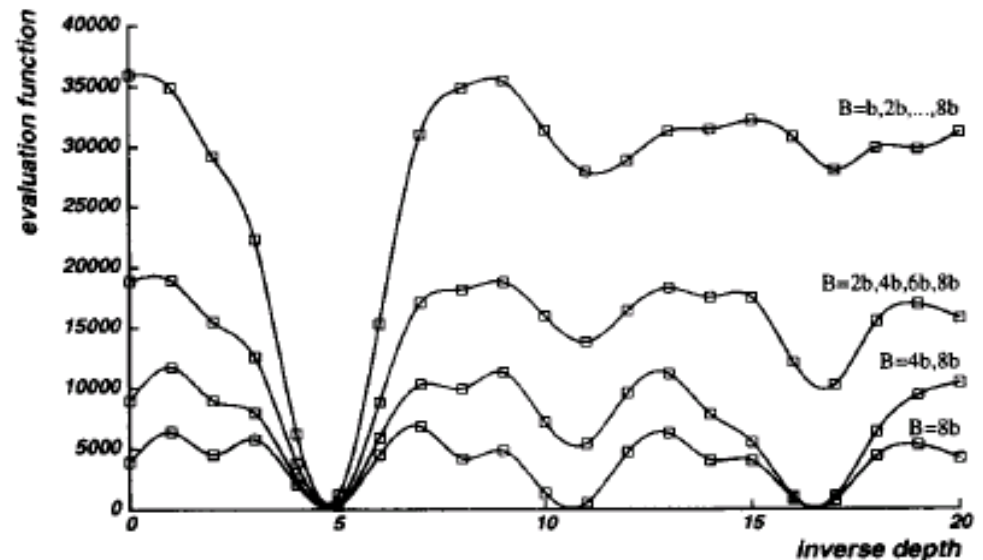
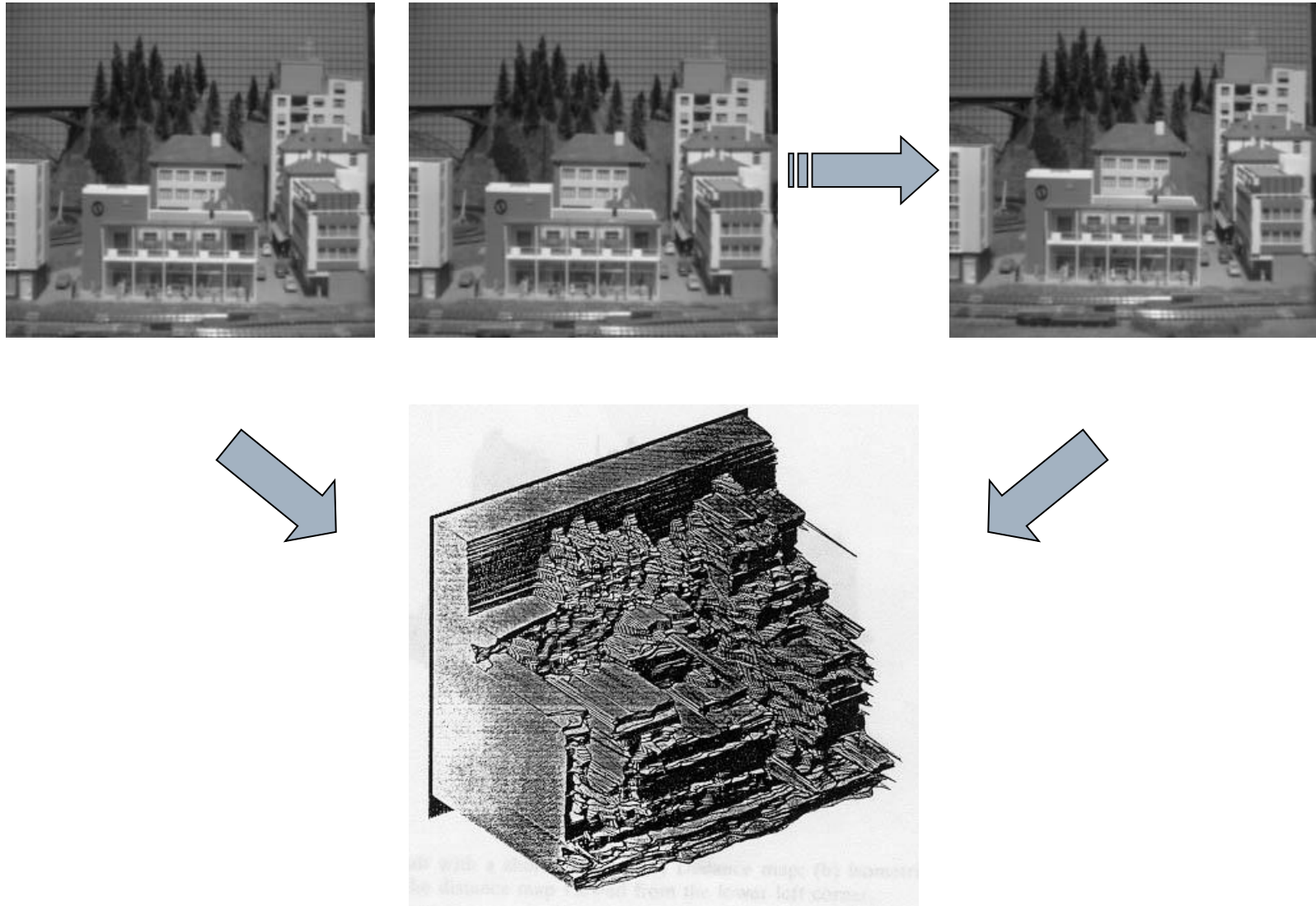


Fig. 7. Combining multiple baseline stereo pairs.

Multiple-baseline stereo results



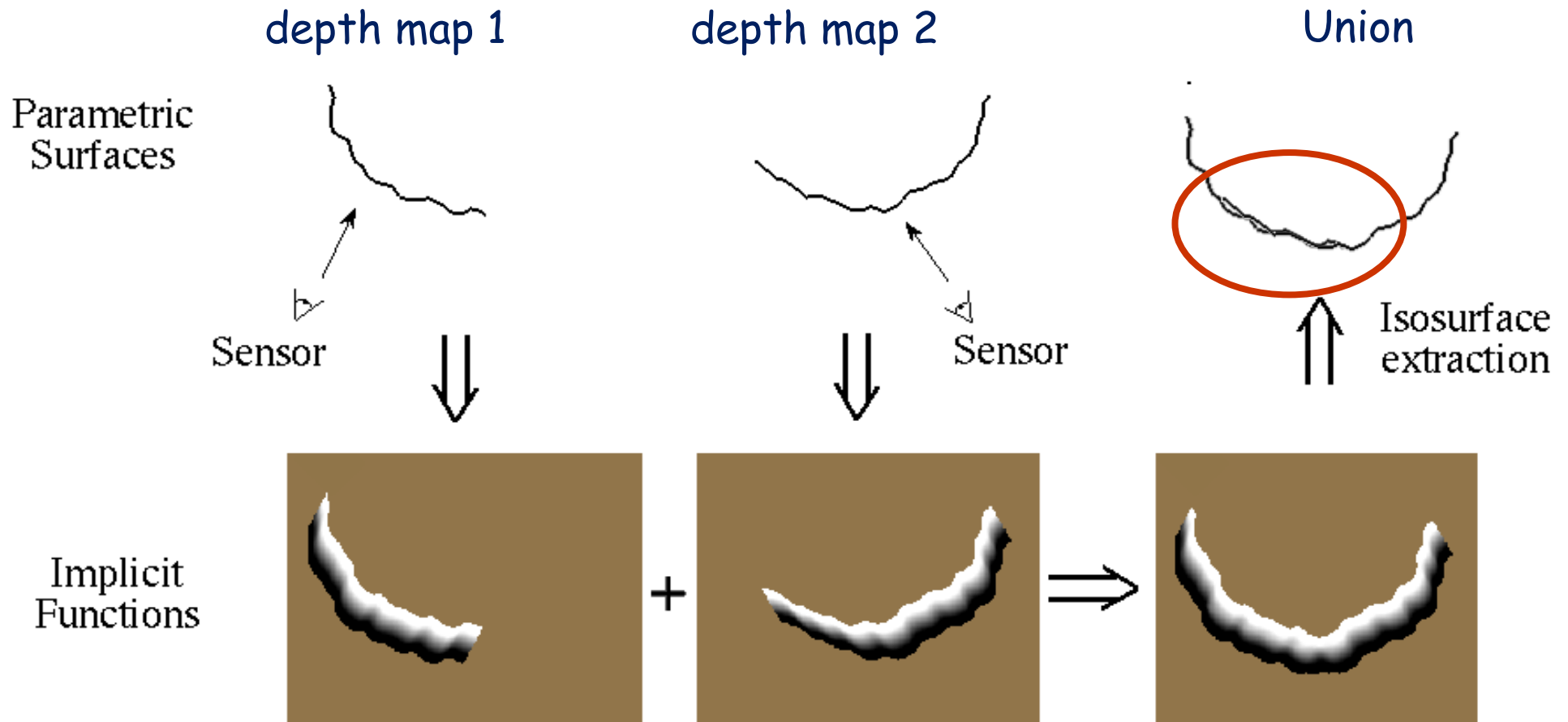
M. Okutomi and T. Kanade, ["A Multiple-Baseline Stereo System,"](#) IEEE Trans. on Pattern Analysis and Machine Intelligence, 15(4):353-363 (1993).

Merging depth maps

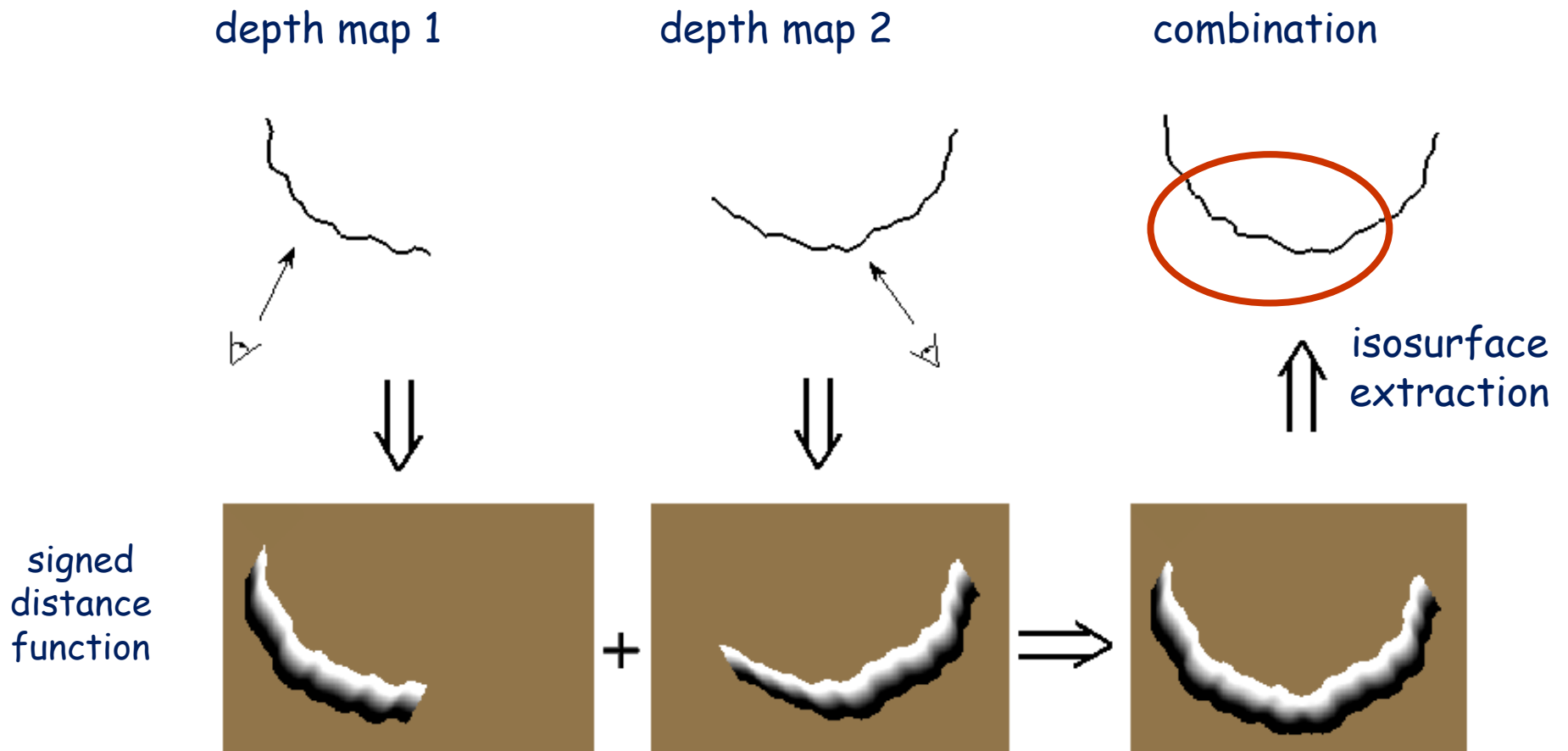
Naïve combination (union) produces artifacts

Better solution: find "average" surface

- Surface that minimizes sum (of squared) distances to the depth maps



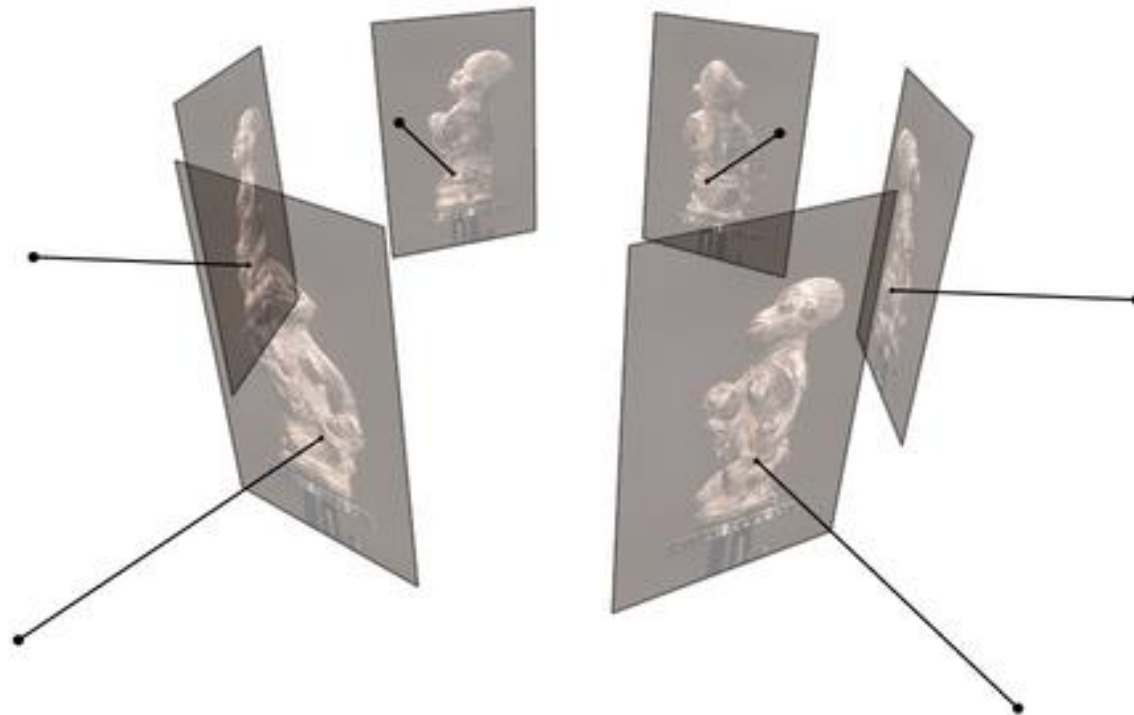
VRIP [Curless & Levoy 1996]



Multi-view Stereo

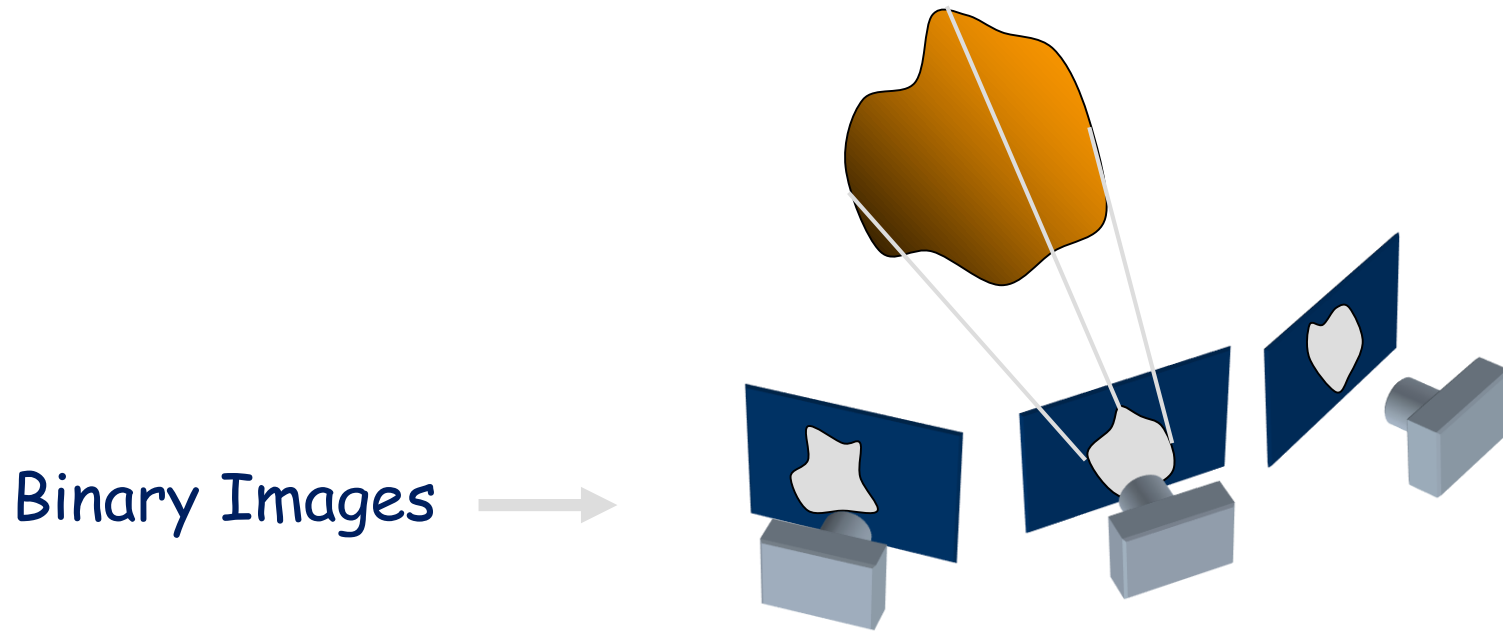
Input: calibrated images from several viewpoints

Output: 3D object model



Figures by Carlos Hernandez

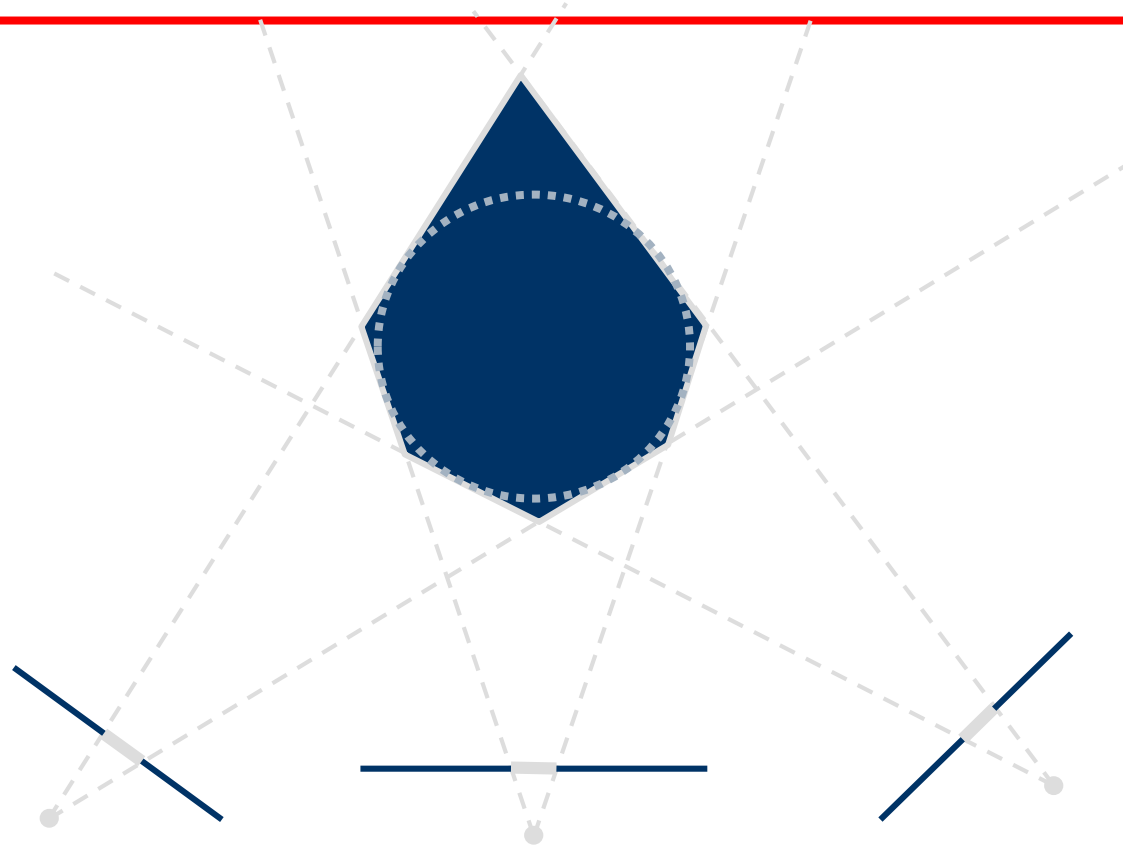
Reconstruction from Silhouettes



Approach:

- *Backproject* each silhouette
 - Intersect backprojected volumes
-

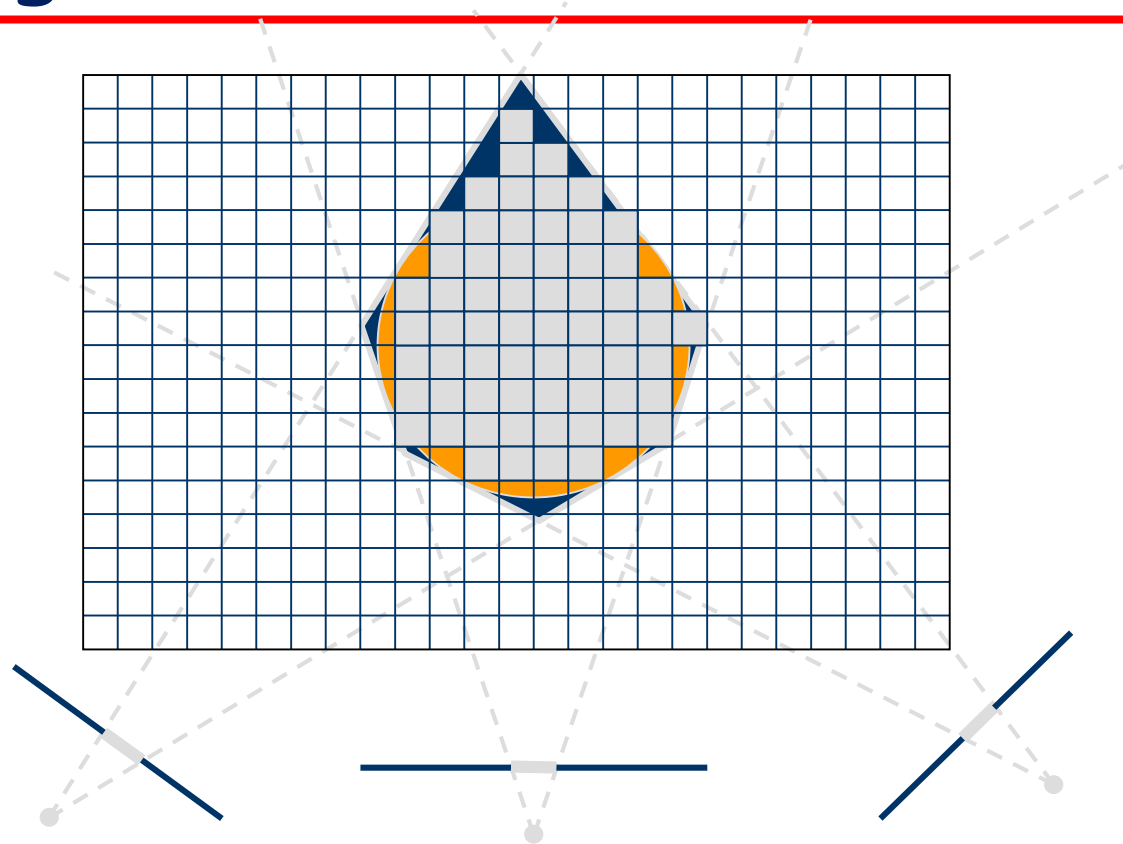
Volume intersection



Reconstruction Contains the True Scene

- But is generally not the same
- In the limit (all views) get *visual hull*
 - ✓ Complement of all lines that don't intersect S

Voxel algorithm for volume intersection

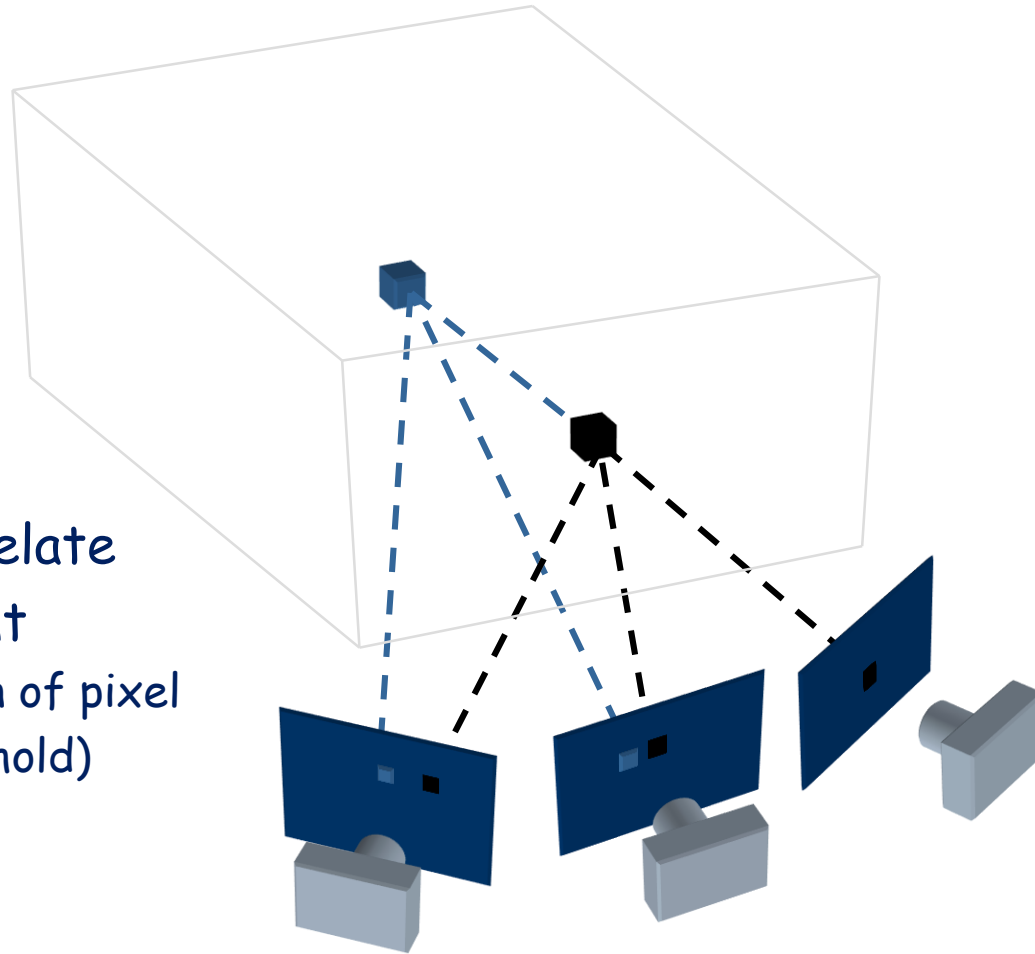


Color voxel black if on silhouette in every image

- $O(N^3)$, for M images, N^3 voxels $O(MN^3)$

Voxel Coloring Approach

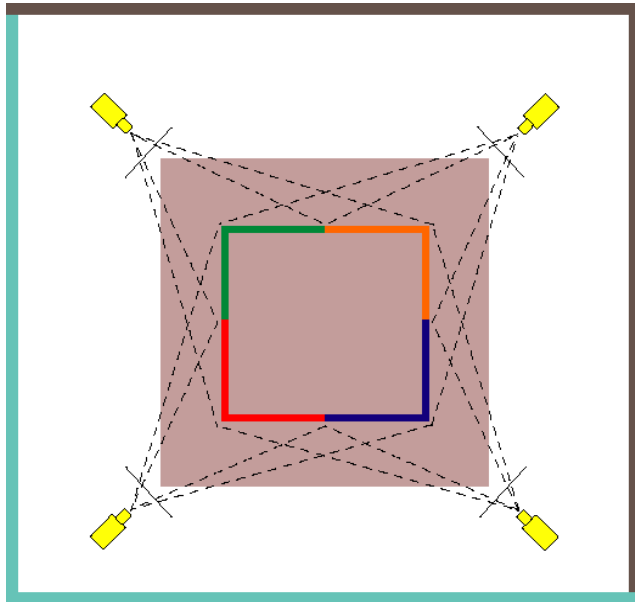
1. Choose voxel
2. Project and correlate
3. Color if consistent
(standard deviation of pixel colors below threshold)



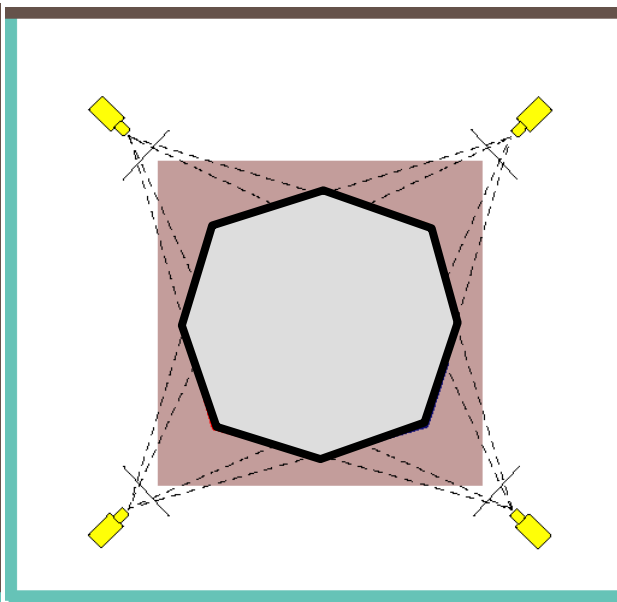
Visibility Problem: in which images is each voxel visible?

Photo-consistency vs. silhouette-consistency

- The *Photo Hull* is the *UNION* of all photo-consistent scenes in V
 - It is a photo-consistent scene reconstruction
 - Tightest possible bound on the true scene



True Scene



Convex Hull

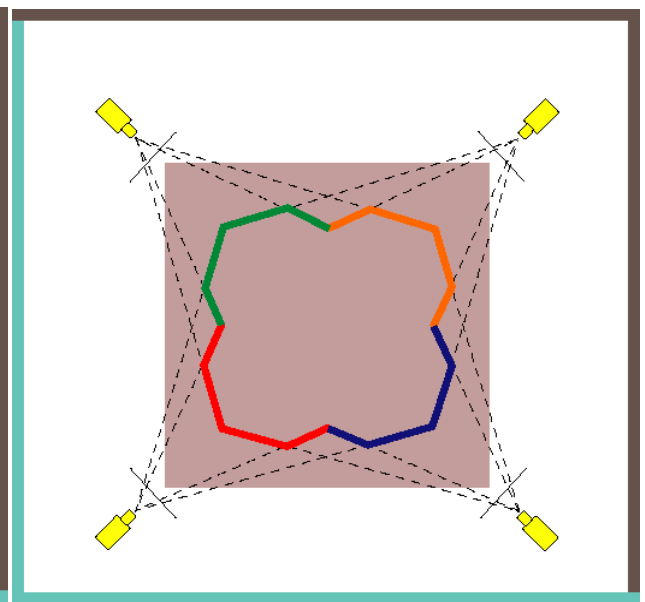


Photo Hull

Carved visual hulls

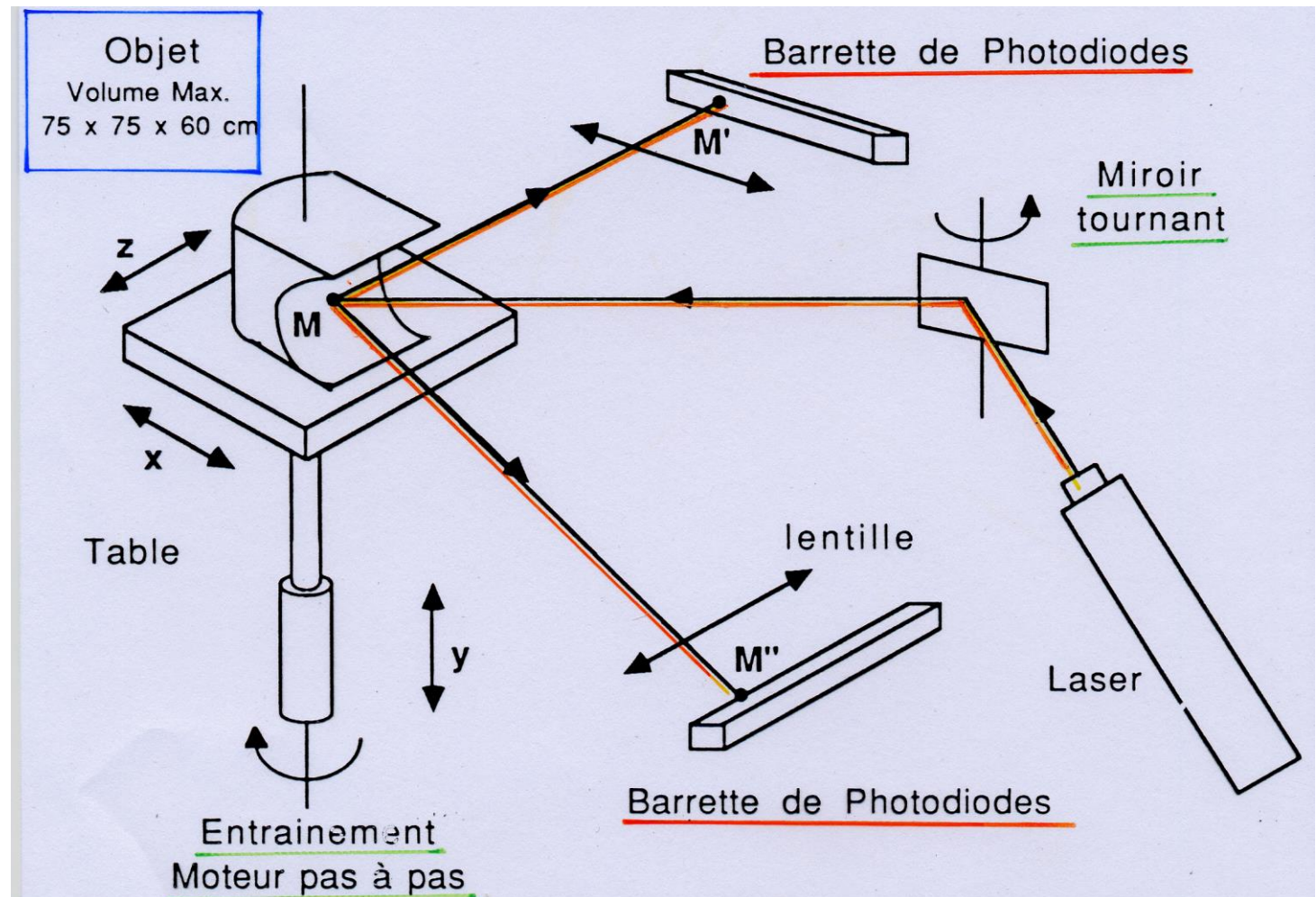
- The visual hull is a good starting point for optimizing photo-consistency
 - Easy to compute
 - Tight outer boundary of the object
 - Parts of the visual hull (rims) already lie on the surface and are already photo-consistent
- Pros
 - Visual hull gives a reasonable initial mesh that can be iteratively deformed
- Cons
 - Need silhouette extraction
 - Have to compute a lot of points that don't lie on the object
 - Finding rims is difficult
- Possible solution: use sparse feature correspondences as initialization

Structured light: point

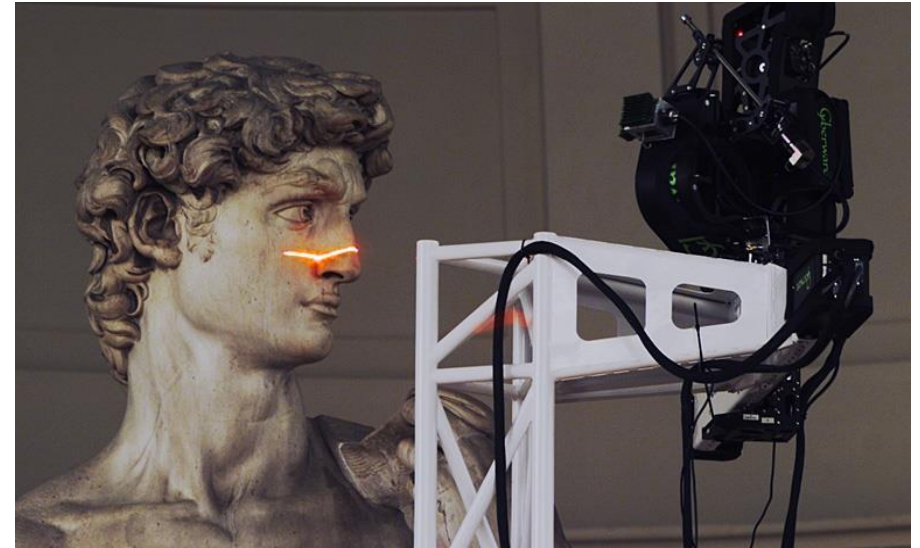
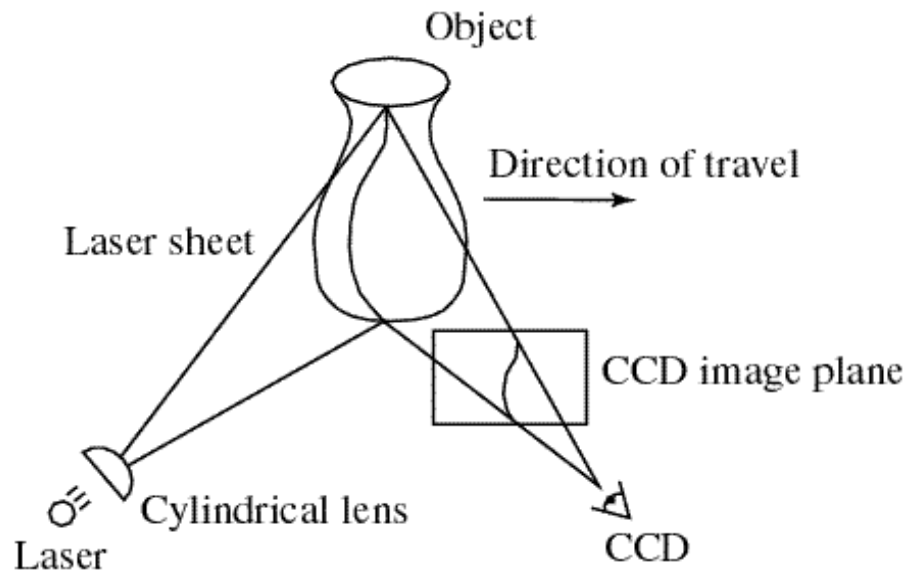
□ Point

□ Plane

□ Grid



Laser scanning



Digital Michelangelo Project
<http://graphics.stanford.edu/projects/mich/>

- Optical triangulation
 - Project a single stripe of laser light
 - Scan it across the surface of the object
 - This is a very precise version of structured light scanning

Laser scanned models



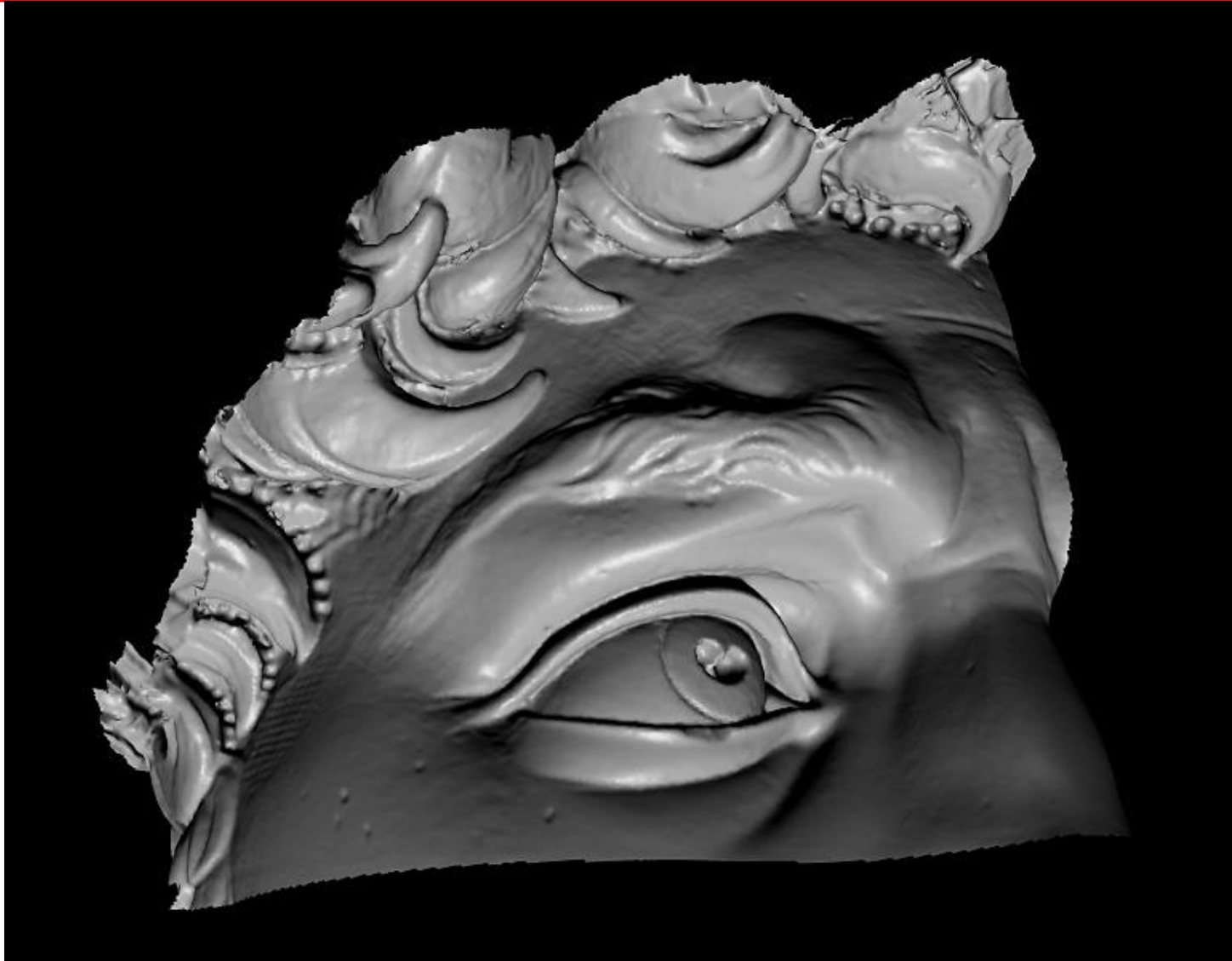
The Digital Michelangelo Project, Levoy et al.

Laser scanned models



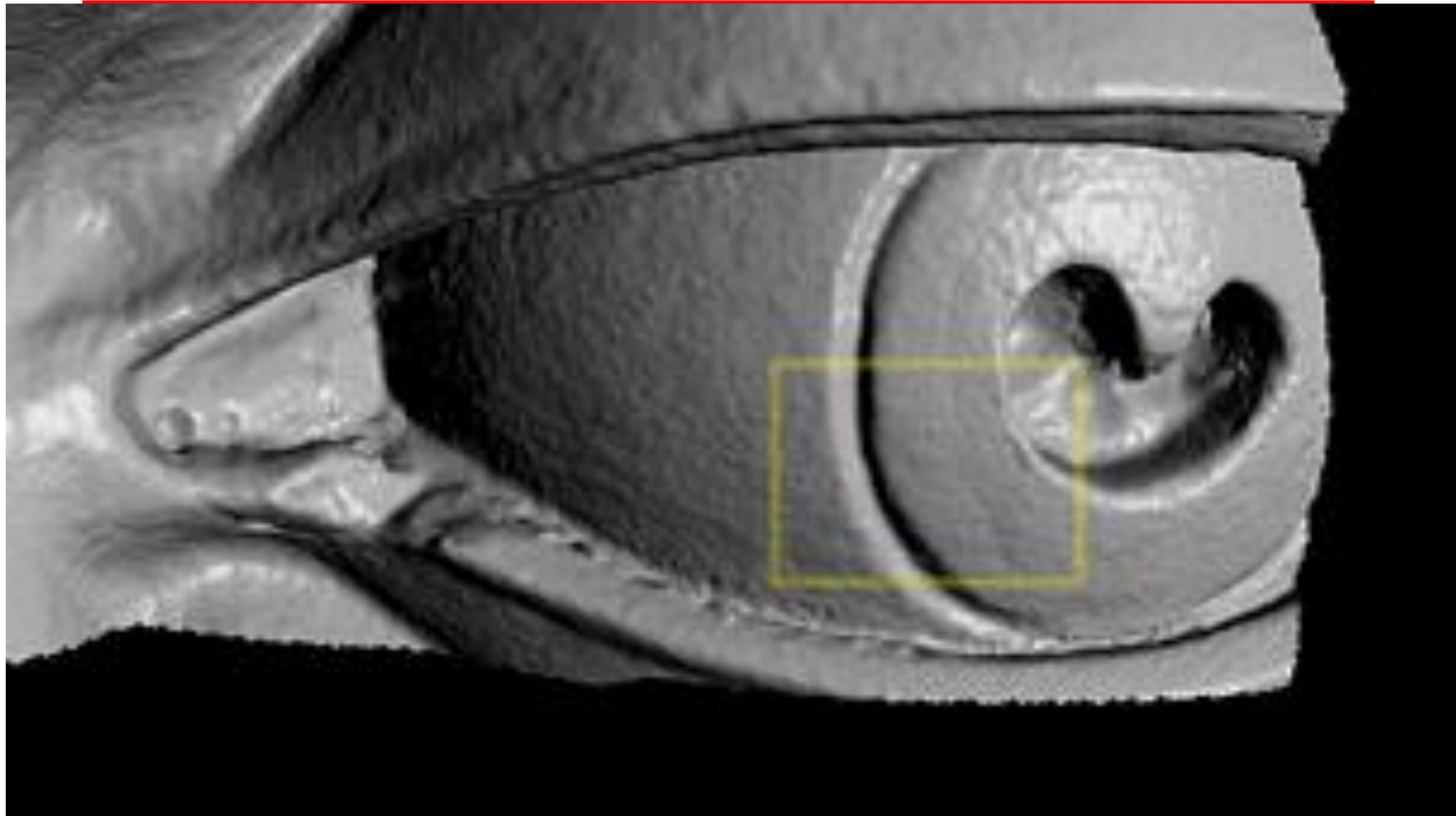
The Digital Michelangelo Project, Levoy et al.

Laser scanned models



The Digital Michelangelo Project, Levoy et al.

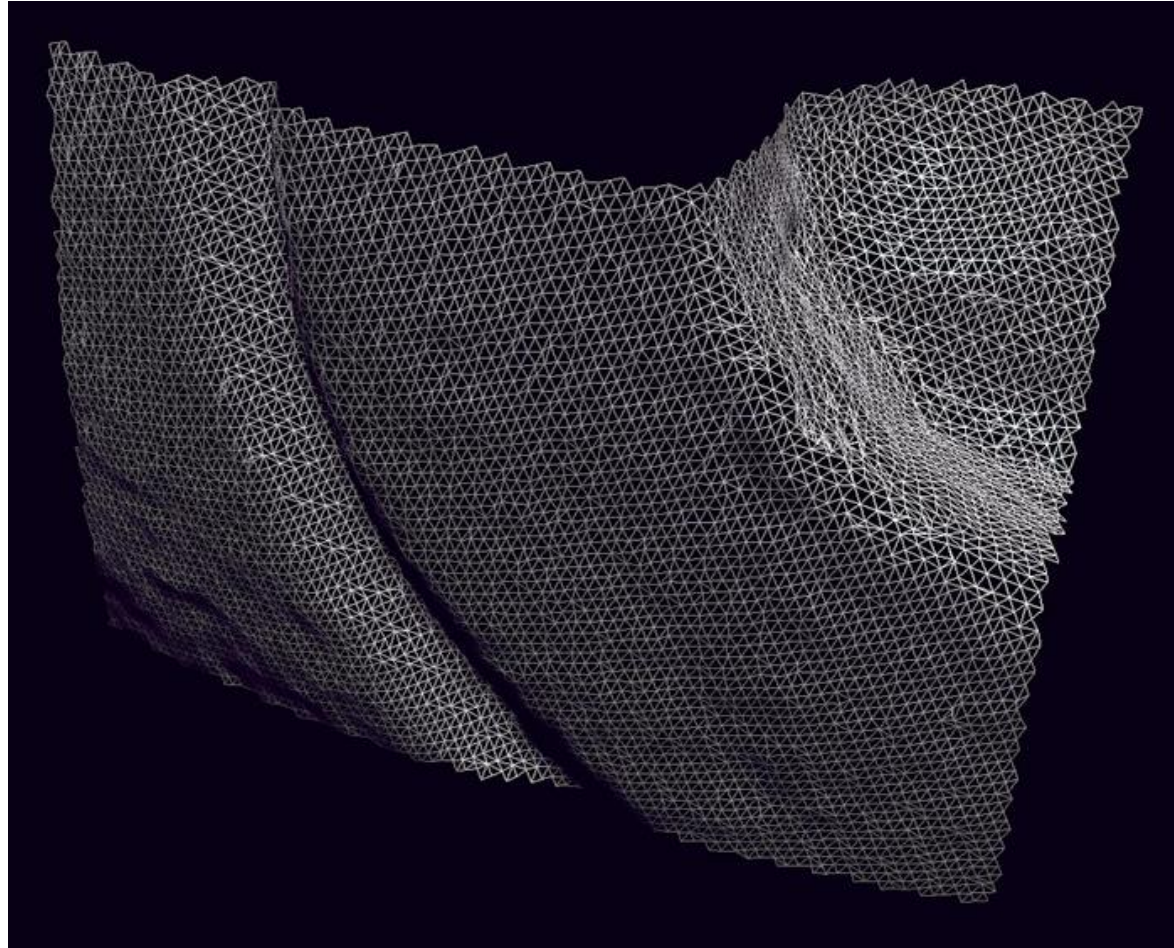
Laser scanned models



The Digital Michelangelo Project, Levoy et al.

Laser scanned models

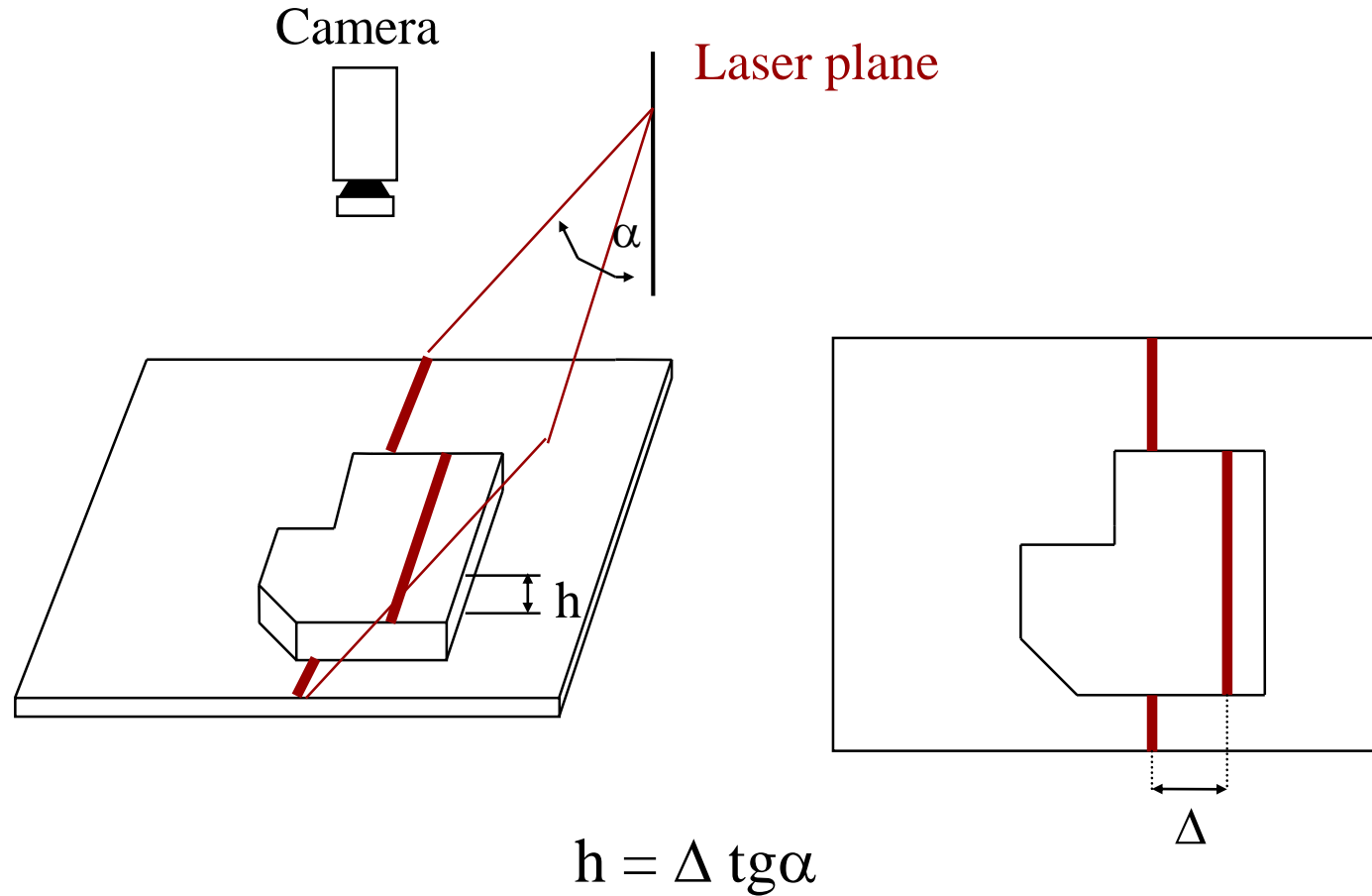
1.0 mm resolution (56 million triangles)



The Digital Michelangelo Project, Levoy et al.

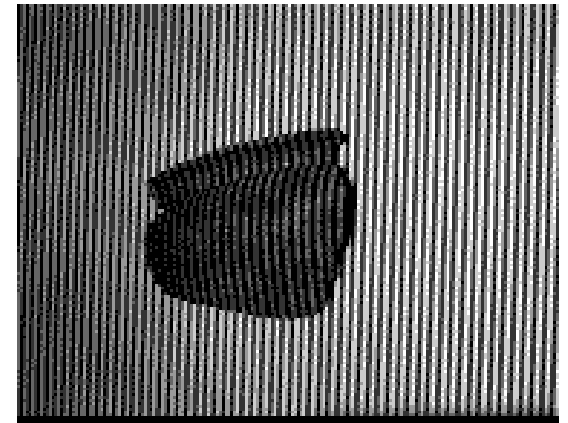
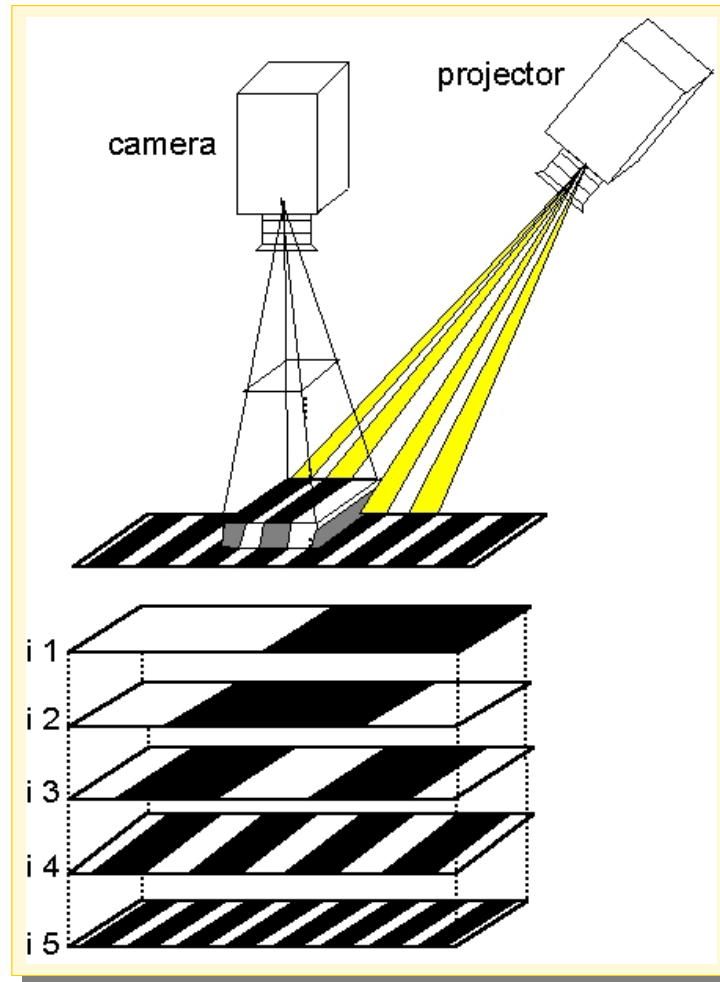
Structured light: plane

- Point
- Plane
- Grid



Structured light: grid

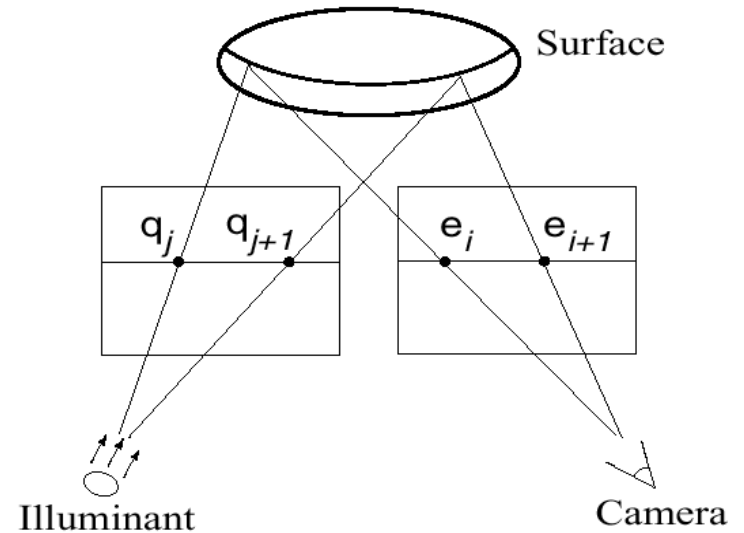
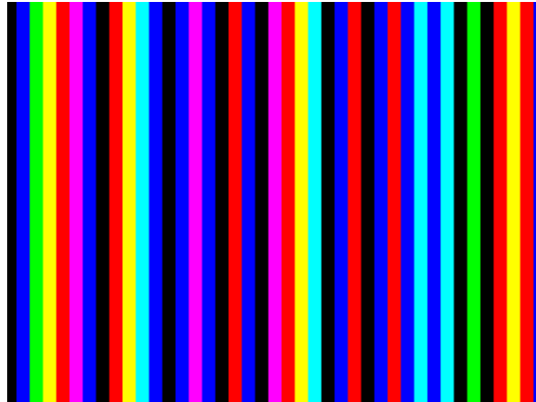
- Point
- Plane
- *Grid*



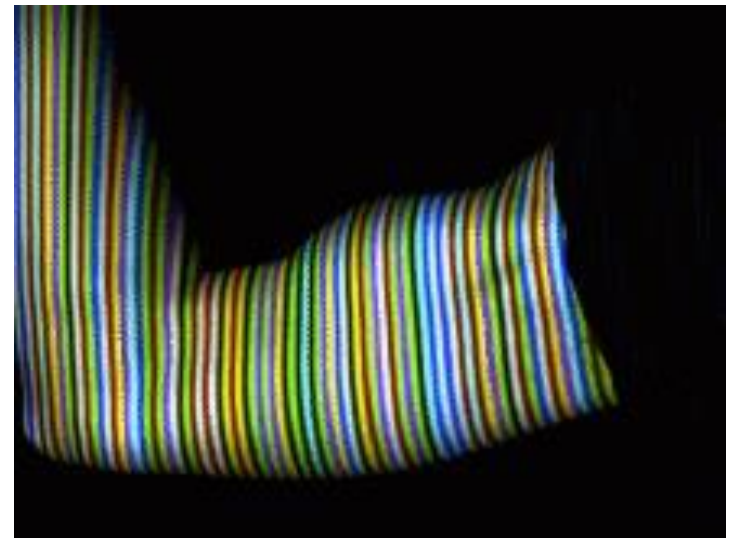
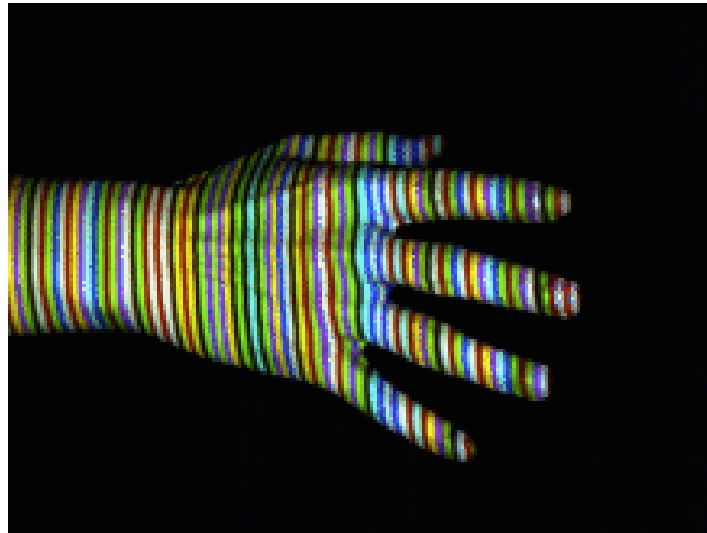
Structured light: plane

□ Point

□ Plane



□ Grid



Single View Metrology

Three-dimensional
reconstruction from **single**
views

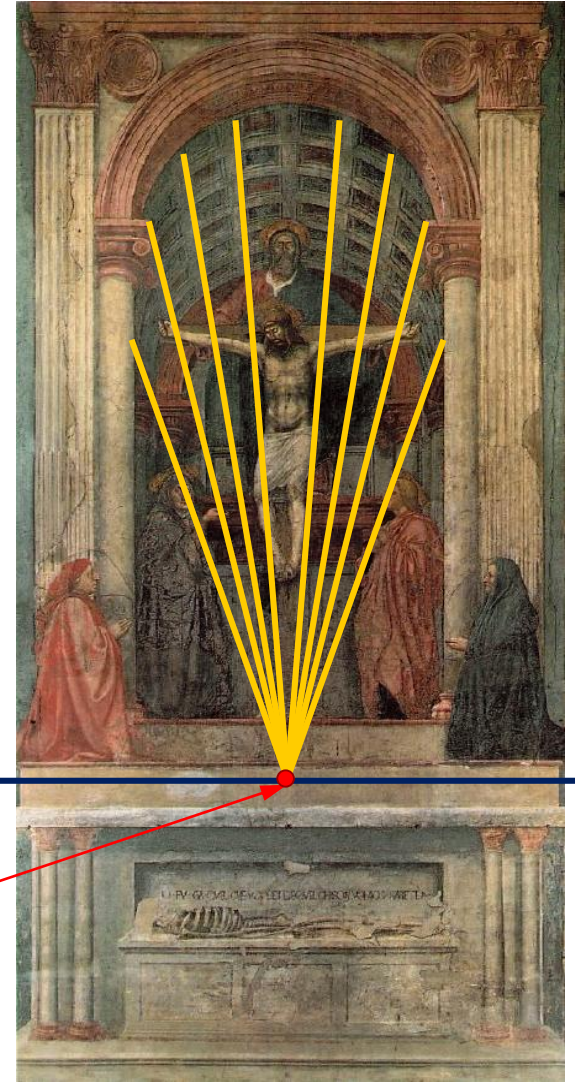
Single-View Reconstruction

- **Geometric cues:** Exploiting vanishing points and vanishing lines
- **Interactive** reconstruction process

Masaccio's *Trinity*

Vanishing line (horizon)

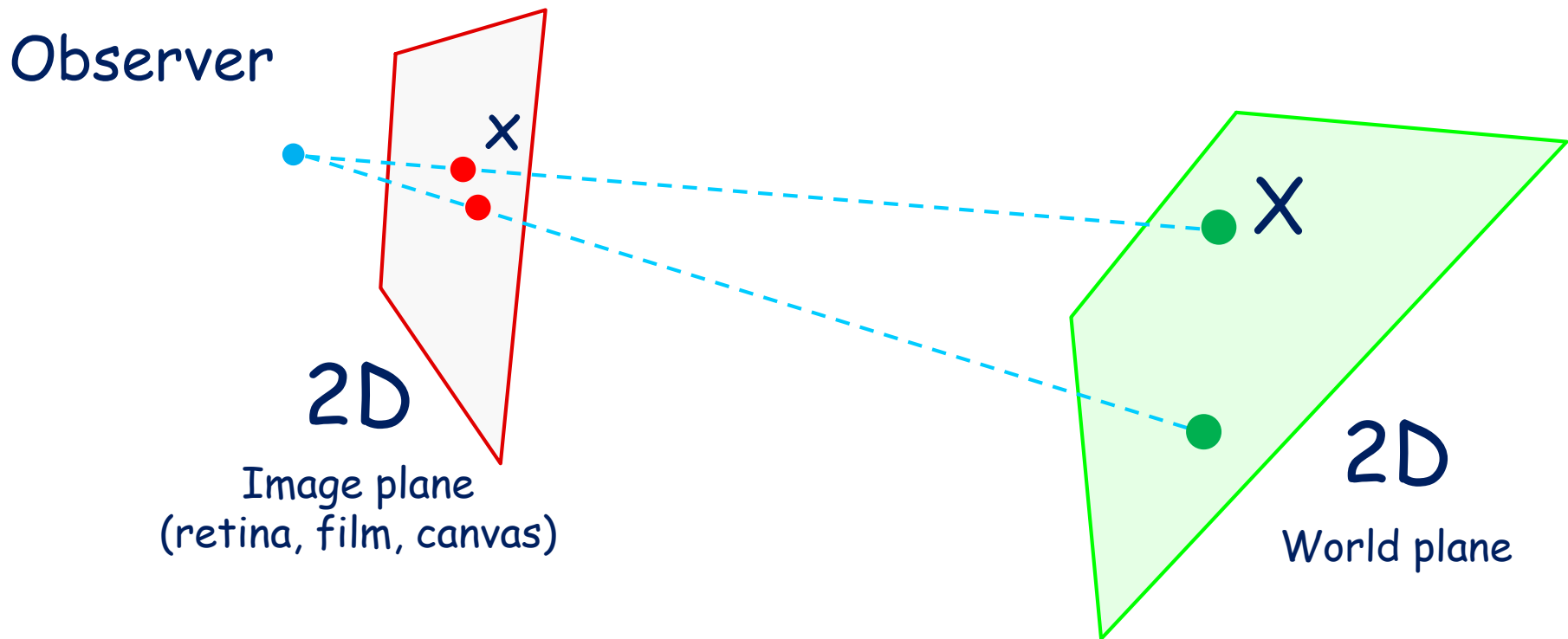
Vanishing point



A special case, planes

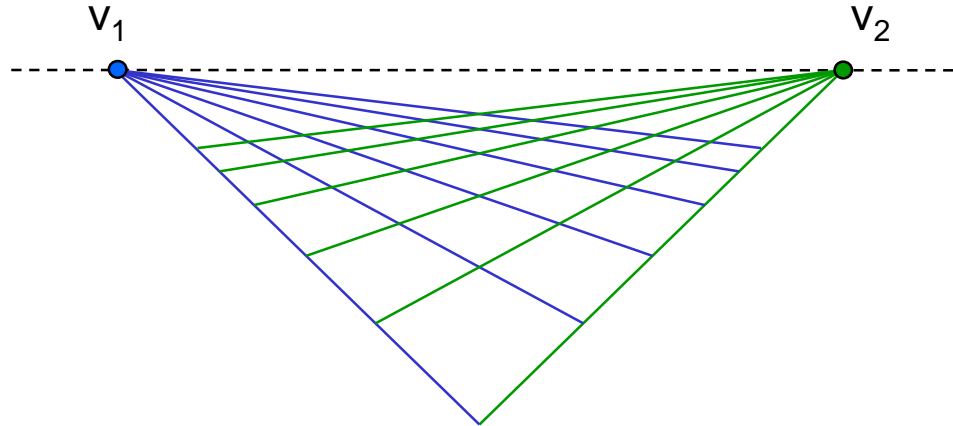
Homography matrix

$$x = HX$$



H : a plane to plane projective transformation

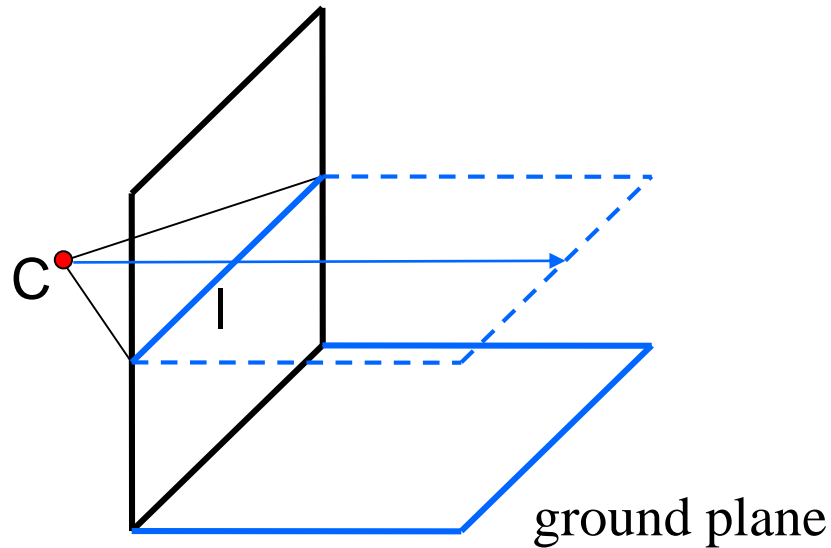
Vanishing lines



■ Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
 - The union of all of vanishing points from lines on the same plane is the vanishing line
 - ✓ For the ground plane, this is called the horizon
-

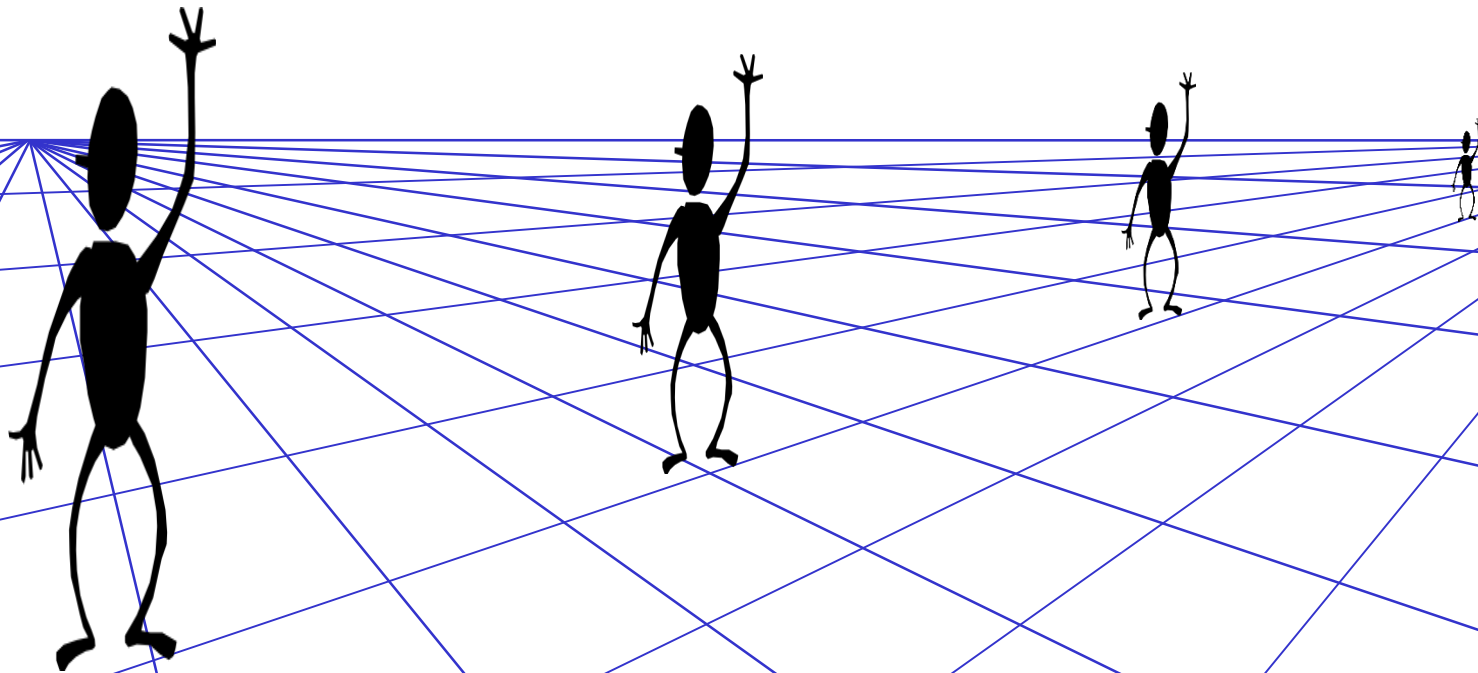
Computing the horizon



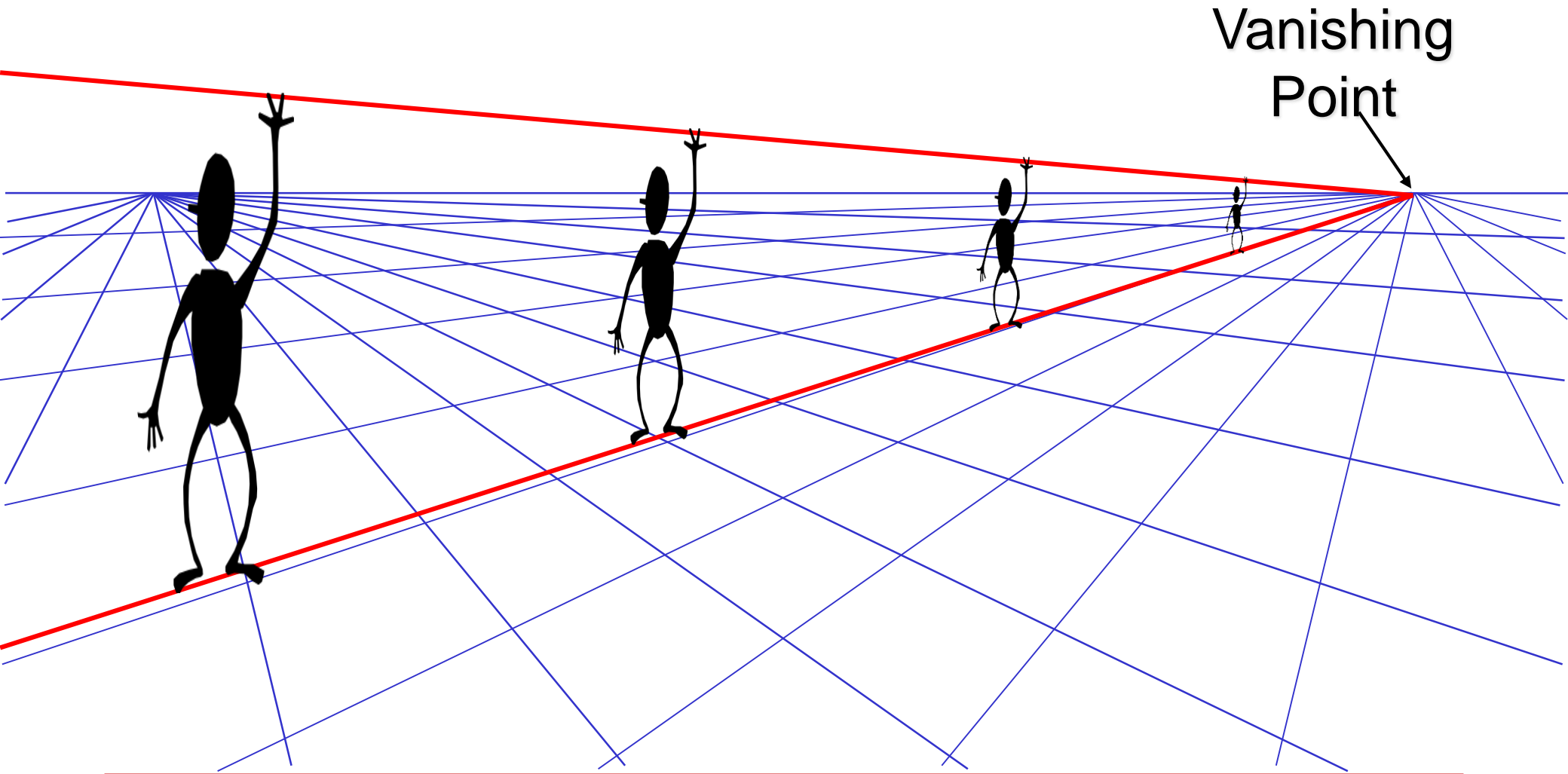
- Properties

- l is intersection of horizontal plane through C with image plane
- Compute l from two sets of parallel lines on ground plane
- All points at same height as C project to l
- Provides way of comparing height of objects in the scene

Are these guys the same height?



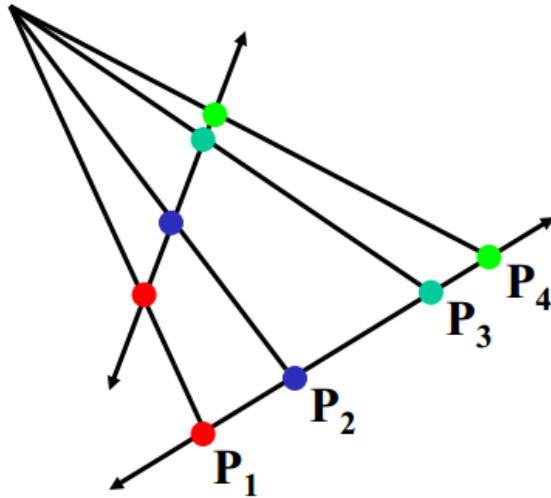
Comparing heights



The cross ratio

- A Projective Invariant
 - Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



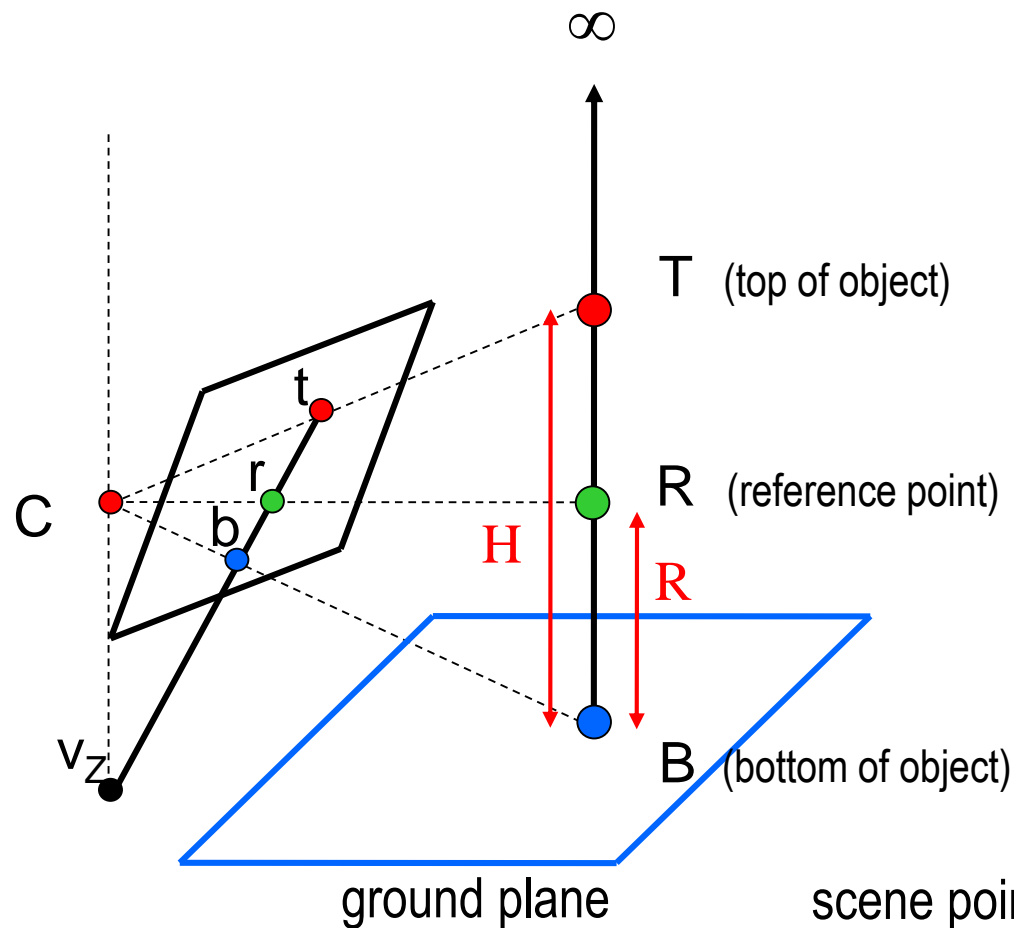
$$\frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|}$$

$$P_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\frac{\|P_1 - P_3\| \|P_4 - P_2\|}{\|P_1 - P_2\| \|P_4 - P_3\|}$$

- Can permute the point ordering
 - $4! = 24$ different orders (but only 6 distinct values)
- This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T} - \mathbf{B}\| \|\infty - \mathbf{R}\|}{\|\mathbf{R} - \mathbf{B}\| \|\infty - \mathbf{T}\|} = \frac{H}{R}$$

scene cross ratio

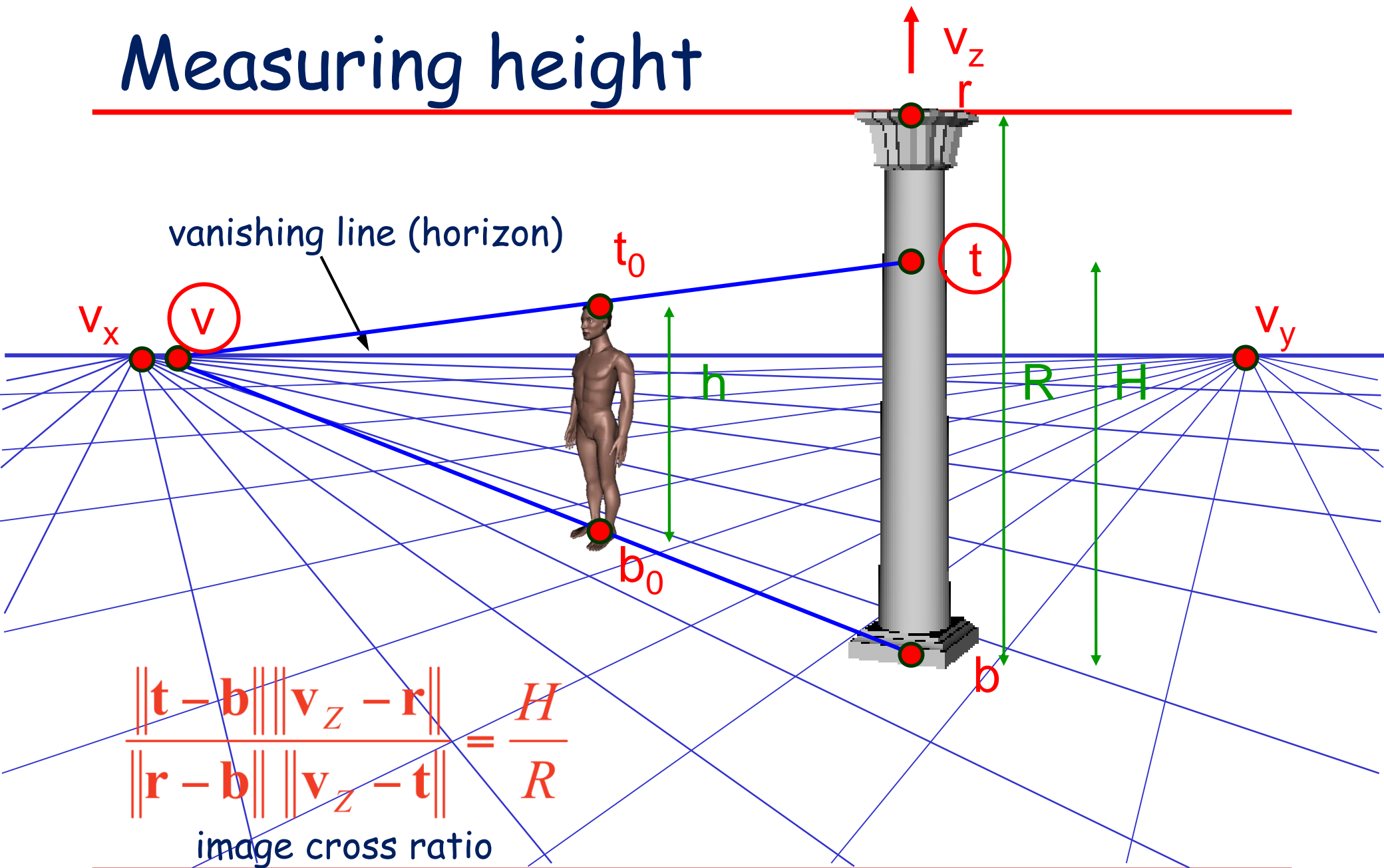
$$\frac{\|\mathbf{t} - \mathbf{b}\| \|\mathbf{v}_z - \mathbf{r}\|}{\|\mathbf{r} - \mathbf{b}\| \|\mathbf{v}_z - \mathbf{t}\|} = \frac{H}{R}$$

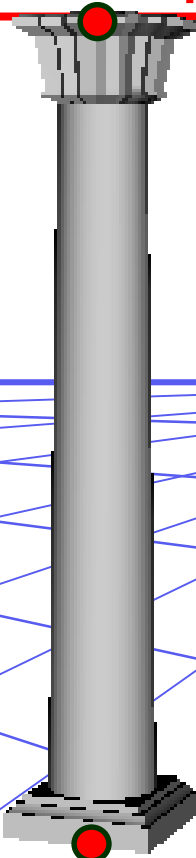
image cross ratio

image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

Measuring height





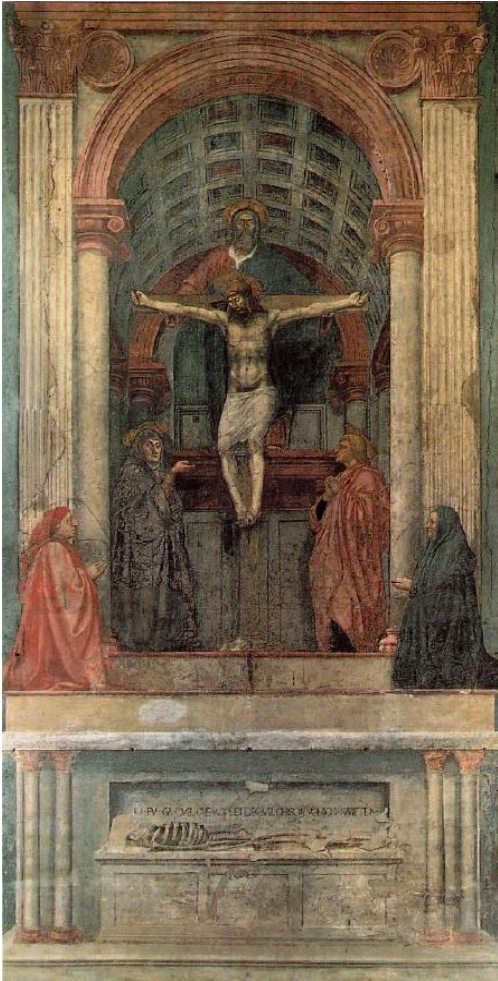
- Here the guy is standing on the box
- Use one side of the box to help find b_0 as shown above

- Here the guy is standing on the box
- Use one side of the box to help find b_0 as shown above

Single-View Metrology

Complete 3D reconstructions
from **single** views

Example: The Virtual Trinity



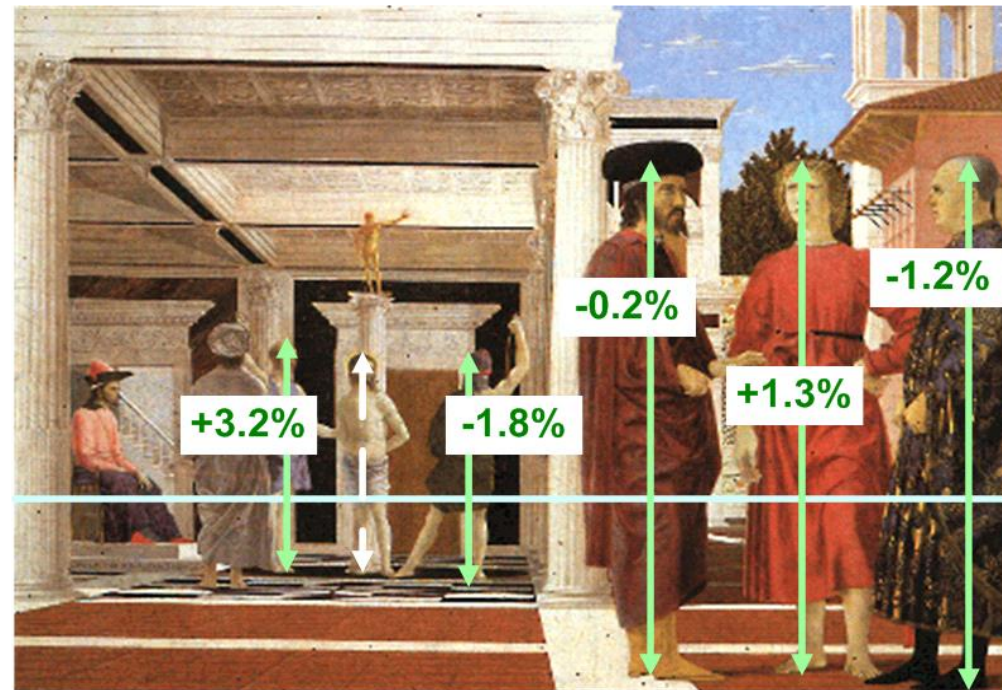
Masaccio, *Trinità*,
1426, Florence



Complete 3D reconstruction

Assessing geometric accuracy

Are the heights of the two groups of people consistent with each other?



Measuring relative heights

Piero della Francesca, *Flagellazione di Cristo*, c.1460, Urbino

Analysing patterns and shapes

What is the shape of the b/w floor pattern?



Homography

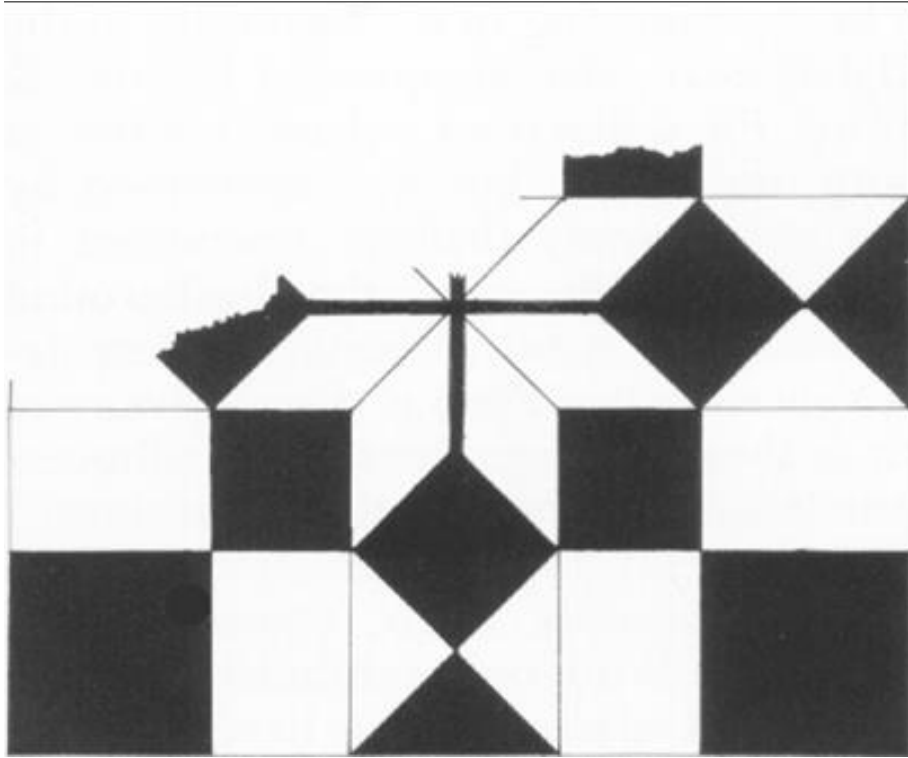


The floor



Automatically rectified floor

Analysing patterns and shapes



From Martin Kemp *The Science of Art*
(*manual reconstruction*)

2 patterns have been discovered !

automatic rectification



Example: The Virtual St. Jerome



Henry V Steenwick,
St. Jerome in His Study,
1630, The Netherlands



Complete 3D reconstruction

Example: The Virtual Music Lesson



J. Vermeer,
The Music Lesson,
1665, London



Reconstructions by Criminisi et al.

A Virtual Museum @ Microsoft

A dive into the paintings third dimension



The museum

Diving into the
paintings



The Trinity
Masaccio



Flagellation
P. della Francesca



St Jerome
H. Steinwick



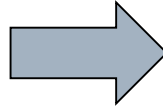
The Music Lesson
J. Vermeer

http://research.microsoft.com/en-us/um/people/antcrim/ACriminisi_3D_Museum.wmv

Why do we perceive depth?



Using more than two images



[Multi-View Stereo for Community Photo Collections](#)

M. Goesele, N. Snavely, B. Curless, H. Hoppe, S. Seitz
Proceedings of [ICCV 2007](#),